Engineering Geology 225 (2017) 42–48

Contents lists available at ScienceDirect

Engineering Geology

journal homepage: www.elsevier.com/locate/enggeo

A peridynamics formulation for quasi-static fracture and contact in rock

Timon Rabczuk[*,a,b], Huilong Ren[b]

[a] Duy Tan University, Institute of Research & Development, 3 Quang Trung, Danang, Vietnam
[b] Institute of Structural Mechanics, Bauhaus-Universität Weimar, Weimar 99423, Germany

ARTICLE INFO

Keywords:
Brittle fracture
Quasi-static
Artificial damping
Contact
Mohr-Coulomb model

ABSTRACT

We present a dual-horizon peridynamics (DH-PD) formulation for fracture in granular and rock-like materials. In contrast to discrete crack methods such as XFEM, DH-PD does not require any representation of the crack surface and criteria to treat complex fracture patterns such as crack branching and coalescence. The crack path is the natural outcome of the simulation. In this manuscript, a new penalty method to model the contact for compressive fractures and constraining the penetration conditions is developed. The new method is applied to several benchmark problems in geomechanics including the four-point shear test, the indirect tensile (Brazilian) test of rock disks with one or multiple initial cracks. By using an appropriate damping coefficient, the quasi-static solution for rock failure is obtained when the dynamic formulation is used. A good agreement is obtained between the results given by DH-PD and those by the experiments.

1. Introduction

The prediction of fracture in rock and rock-like materials is still challenging due to the complicated behavior of those materials which include anisotropy, cracks, pores, pre-existing joints and faults, to name a few. Dealing with complex fracture patterns and predicting the strength of rock materials is of great importance for numerous engineering applications. In order to determine the rock fracture-toughness for example, the Brazilian splitting test is commonly adopted (Guo et al., 1993). However, besides advances in computational methods for fracture, there is still an urgent need for reliable computational methods which can handle the complex fracture behavior of rock.

The numerical modeling of fracture patterns in rock has been an interesting and active field in recent years since modeling can provide complementary guidance, quantitative assessment and predictive information which can not be achieved by traditional empirical ways alone. A wide variety of methods have been proposed in the past as outlined in recent surveys (Jing, 2003). These methods are classified into the family of continuum approaches, discrete crack approaches and discontinuum approaches according to their capability in simulating the mechanical behavior of the problem. Discontinuum approaches such as the discrete element method (DEM) and discontinuous deformation analysis (DDA) (Shi and Goodman, 1985; Zhu et al., 2015) are effective in approximating discontinuous system such as fractured rock masses. The most popular continuous methods to fracture is probably the finite element method (FEM) which employs regularization techniques such as the screened Poisson equation in order to restore the well-posedness of the boundary value problem (Areias et al., 2016). Popular discrete fracture approaches include the extended finite element method (XFEM) (Belytschko and Black, 1999; Moes et al., 1999), numerical manifold method (NMM), efficient remeshing techniques (Areias et al., 2013, 2015) and certain meshless methods (Amiri et al., 2014; Rabczuk and Belytschko, 2004a; Zhuang and Augarde, 2010). While most of those methods have been applied outside the area of geomechanics, there are some interesting extensions of those method to model fracture in geological materials. The numerical manifold method has been used to model the failure of rock slope (Zhang et al., 2007). Wu et al. (2013) modeled the cracking processes of rectangular rock mass under uniaxial compression with the enriched numerical manifold method. The fracture propagation modeled by finite element method often heavily depends on the mesh alignment, where other techniques such as continuous damage method, or adaptive mesh refinement or cohesive zone method (Elices et al., 2002) are required.

Apart from these conventional methods aforementioned, multiscale methods (Yang et al., 2015) and fine-scale approaches based on e.g. molecular dynamics (MD) (Budarapu et al., 2014, 2015) are adopted to model fractures. Yang et al. (2015) combined the reproducing kernel particle method and molecular static approach to model static crack growth. Budarapu et al. (2014) proposed an adaptive atomistic-continuum numerical method for quasi-static crack growth. The continuum region is modeled with the phantom node method while the crack tip region is described by an MD approach. Another 'unconventional' approach to fracture is Peridynamics (PD) which shows similarities to MD and also certain meshfree approaches. PD has recently attracted

* Corresponding author.
E-mail address: timon.rabczuk@uni-weimar.de (T. Rabczuk).

http://dx.doi.org/10.1016/j.enggeo.2017.05.001
Received 24 November 2016; Received in revised form 16 April 2017; Accepted 2 May 2017
Available online 05 May 2017

broad interests since crack nucleation and propagation is the natural outcome of the simulation. In contrast to conventional methods such as XFEM, it does not require any description of the crack topology and no cracking criterion which determines the direction of fracture propagation. The cracking criterion is incorporated into the PD theory. In PD, the equation of motion is formulated in an integral form rather than the partial differential form. The material point interacts with other material points in its horizon, a domain associated to the point. The horizon is often represented by circular area (2D) or spherical region (3D) with a radius. There are mainly 3 types of PD formulations, i.e. bond based peridynamics (BB-PD) (Silling, 2000), ordinary state based peridynamics (OSB-PD) and non-ordinary state based peridynamics (NOSB-PD). In BB-PD, the bonds act like independent springs. One limitation of BB-PD is that only the isotropic material with Poisson's ratio of 1/3 in 2D or 1/4 in 3D can be modeled. In order to eliminate the Poisson's ratio restriction, the state based peridynamics (Silling et al., 2007) is proposed, where "state" means the bond deformation depends not only on the deformation of the bond itself, but also on collective deformation of other bonds. The SB-PD was extended into two types, namely the OSB-PD and the NOSB-PD. NOSB-PD is capable of simulating the material with advanced constitute models (Foster et al., 2010). In addition, in order to improve the efficiency of peridynamics in fracture modeling, much work was performed on the coupling of peridynamics with the continuum methods (Lubineau et al., 2012). The coupling method enables to model the fracture domain with PD while other domain modeled with continuum method. In this method, the PD is discretized with finite element instead of particles, and the local integration matrix is assembled.

In this paper, the dual-horizon peridynamics (DH-PD) method (Ren et al., 2016a) is extended and applied for analyzing crack propagation in rock. The key novelty is that it can capture complex fracture including micro-cracking, etc. more naturally as pointed out in the previous paragraph. In contrast to the traditional PD approach, DH-PD allows varying the horizon sizes based on the material point's volume and the arrangement of nearby particles and is therefore computationally cheaper. The derivation of the current peridynamics is based on the Newton's third law, and other techniques such as Taylor expansion and variational calculus are not required. The conservations of momentum, angular momentum and energy are naturally satisfied. In order to explore the potential of DH-PD in quasi-static crack simulation, the artificial damping force is added in the governing equation. In addition, a simple contact algorithm based on the non-linear damping contact model is used to account for compressive load conditions. The bucket search algorithm is employed to identify contact between particles with high efficiency.

This paper is structured as follows. In Section 2, DH-PD is described in detail. In Section 3, we present a penalty method to take into account the contact phenomenon in peridynamics. In Section 4, the artificial damping coefficient is introduced to enhance the quasi-static simulation using peridynamics. In Section 5, three numerical examples including the four-point shear test, the square with multiple cracks and the Brazilian splitting test are presented to validate the method. Finally, we conclude the findings in this study and discuss the issues need to be addressed for future development.

2. Review of peridynamics

Consider a solid in the initial and current configuration as shown in Fig. 1. Let $\mathbf{x}$ and $\mathbf{x}'$be the material coordinates in initial configuration Ω_0; and $\mathbf{y}$:=$\mathbf{y}(\mathbf{x},t)$ and $\mathbf{y}'$:=$\mathbf{y}(\mathbf{x}',t)$are the spatial coordinates in the current configuration Ω_t for $\mathbf{x}$ and $\mathbf{x}'$, respectively; $\boldsymbol{\xi}$:=$\mathbf{x}'-\mathbf{x}$ is initial bond vector, the relative distance vector between $\mathbf{x}$ and $\mathbf{x}'$; $\mathbf{u}$:=$\mathbf{u}(\mathbf{x},t)$ and $\mathbf{u}'$:=$\mathbf{u}(\mathbf{x}',t)$ are the displacement vectors for $\mathbf{x}$ and $\mathbf{x}'$, respectively; $\boldsymbol{\eta}$:=$\mathbf{u}'-\mathbf{u}$ is the relative displacement vector for bond $\boldsymbol{\xi}$; $\mathbf{y}\langle\boldsymbol{\xi}\rangle$:=$\mathbf{y}(\mathbf{x}',t)-\mathbf{y}(\mathbf{x},t)=\boldsymbol{\xi}+\boldsymbol{\eta}$ is the current bond vector for bond $\boldsymbol{\xi}$. Since the bond $\boldsymbol{\xi}$ starts from $\mathbf{x}$ to $\mathbf{x}'$, an alias $\mathbf{x}\,\mathbf{x}'$ is used to denote $\boldsymbol{\xi}$. Obviously, $\mathbf{x}'\mathbf{x}$ is in

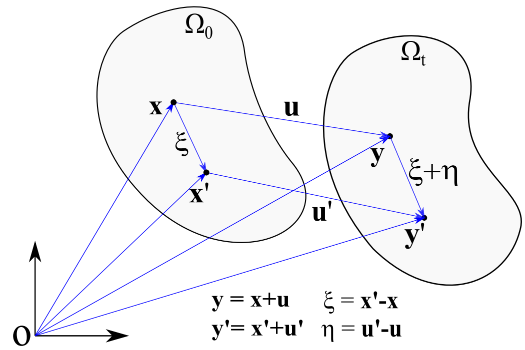

Fig. 1. The configuration for deformed body.

opposite direction of $\mathbf{x}\,\mathbf{x}'$. Let $\mathbf{f}_{\mathbf{x}\mathbf{x}'}:=\mathbf{f}_{\mathbf{x}\mathbf{x}'}(\boldsymbol{\eta},\boldsymbol{\xi})$ denote the force vector density acting on point $\mathbf{x}$ due to bond $\mathbf{x}\,\mathbf{x}'$; based on Newton's third law, $\mathbf{x}'$ undertakes a reaction force $-\mathbf{f}_{\mathbf{x}\mathbf{x}'}$.

As the peridynamics with constant horizon can be viewed as a special case of the DH-PD (Ren et al., 2016a), we here mainly focus on the dual-horizon peridynamics. The horizon for any point $\mathbf{x}$ in peridynamics is defined as

$$H_{\mathbf{x}}=\{\mathbf{x}'|\|\mathbf{x}'-\mathbf{x}\|\le\delta_{\mathbf{x}}\} \tag{1}$$

where $\|\cdot\|$ denotes the Euclidean distance and $\delta_{\mathbf{x}}$ is particle $\mathbf{x}$'s horizon radius. Any point $\mathbf{x}'$ in $H_{\mathbf{x}}$ forms bond $\mathbf{x}\,\mathbf{x}'$. In this sense, the horizon $H_{\mathbf{x}}$ is the union of all bonds starting from $\mathbf{x}$. The dual-horizon is defined as the dual part of horizon

$$H'_{\mathbf{x}}=\{\mathbf{x}'|\mathbf{x}\in H_{\mathbf{x}'}\} \tag{2}$$

Any point $\mathbf{x}'$ in $H'_{\mathbf{x}}$ forms dual-bond $\mathbf{x}'\mathbf{x}$. In this sense, the dual-horizon $H'_{\mathbf{x}}$ is the union of all bonds starting from $\mathbf{x}'$. One particle's bond is another particle's dual-bond, and vice-verse.

For any point $\mathbf{x}$ with density ρ and associated volume $\Delta V_{\mathbf{x}}$, it mainly undertakes three types of forces: (a) the inertia force $\rho\ddot{\mathbf{u}}(\mathbf{x},t)\Delta V_{\mathbf{x}}$, (b) body force $\mathbf{b}(\mathbf{x},t)\Delta V_{\mathbf{x}}$, and (c) internal forces. The internal forces comprise of two parts: the direct forces from $H_{\mathbf{x}}$, and the reaction forces from $H'_{\mathbf{x}}$. Combining all forces for particle $\mathbf{x}$ leads to the equation of motion for dual-horizon peridynamics,

$$\rho\ddot{\mathbf{u}}(\mathbf{x},t)=\sum_{H_{\mathbf{x}}}\mathbf{f}_{\mathbf{x}\mathbf{x}'}(\boldsymbol{\eta},\boldsymbol{\xi})\Delta V_{\mathbf{x}'}-\sum_{H'_{\mathbf{x}}}\mathbf{f}_{\mathbf{x}'\mathbf{x}}(-\boldsymbol{\eta},-\boldsymbol{\xi})\Delta V_{\mathbf{x}'}+\mathbf{b}(\mathbf{x},t)$$

or integral form

$$\rho\ddot{\mathbf{u}}(\mathbf{x},t)=\int_{H_{\mathbf{x}}}\mathbf{f}_{\mathbf{x}\mathbf{x}'}(\boldsymbol{\eta},\boldsymbol{\xi})\,dV_{\mathbf{x}'}-\int_{H'_{\mathbf{x}}}\mathbf{f}_{\mathbf{x}'\mathbf{x}}(-\boldsymbol{\eta},-\boldsymbol{\xi})\,dV_{\mathbf{x}'}+\mathbf{b}(\mathbf{x},t).$$

When all horizons are constant, $H'_{\mathbf{x}}=H_{\mathbf{x}}$ leads to the traditional horizon-constant peridynamics.

For bond-based peridynamics, the bond force is

$$\mathbf{f}_{\mathbf{x}\mathbf{x}'}=C(\delta_{\mathbf{x}})\cdot s_{\mathbf{x}\mathbf{x}'}\cdot\frac{\boldsymbol{\eta}+\boldsymbol{\xi}}{\|\boldsymbol{\eta}+\boldsymbol{\xi}\|},\qquad\forall\,\mathbf{x}'\in H_{\mathbf{x}},$$

$$\mathbf{f}_{\mathbf{x}'\mathbf{x}}=C(\delta_{\mathbf{x}'})\cdot s_{\mathbf{x}\mathbf{x}'}\cdot\frac{-(\boldsymbol{\eta}+\boldsymbol{\xi})}{\|\boldsymbol{\eta}+\boldsymbol{\xi}\|},\qquad\forall\,\mathbf{x}'\in H'_{\mathbf{x}},$$

where the micro-modulus $C(\delta)$ and critical stretch $s_0(\delta)$ in 3 dimensions (Silling and Askari, 2005) are given by

$$C(\delta)=\frac{3E}{\pi\delta^4(1-2\nu)},\qquad s_0(\delta)=\sqrt{\frac{5G_0}{6E\delta}}, \tag{3}$$

where E and ν are the elastic modulus and Poisson ratio, G_0 is the critical energy release rate. An auxiliary scalar μ is used to denote the damage state of a bond, i.e. broken bond ($\mu=0$) and unbroken bond ($\mu=1$). For BB-PD, when the bond stretch reached the critical stretch $s_0(\delta)$, μ is set to 0 irreversibly. The damage for a material point $\mathbf{x}$ is calculated by