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# Modelling hydraulic fractures in porous media using flow cohesive interface elements

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ABSTRACT

This paper revisits the problem of computational modelling of a fluid-driven fracture propagating in a permeable porous medium using zero-thickness flow cohesive interface elements. Both cases of continuous and discontinuous pressure field across the fractures are implemented in a unified formulation. The paper provides computational aspects of hydraulic fracture modelling such as mesh generation, execution time, convergence and numerical integration issues. We show that Newton-Cotes quadrature must be used for quadratic flow cohesive interface elements at least for the presented problems. Our simulations exhibit the so-called intermittent crack tip advancement as recently confirmed in the literature. This paper is addressed to researchers who would like to have a quick working implementation of the zero-thickness flow cohesive interface elements for simulating hydraulic fracturing processes with finite elements.

## 1. Introduction

Natural gas production from hydrocarbon-rich shale formations, known as shale gas, is one of the most rapidly expanding trends in onshore domestic oil and gas exploration and production today. Hydraulic fracturing (HF) or fracking is a common technique for enhancing hydrocarbon production since the late 1940s because of the low permeability of these formations and the low conductivity of the natural fracture networks (Adachi et al., 2007). Hydraulic fracturing can be broadly defined as the process by which a fracture initiates and propagates, in a compressively prestressed solid media (usually sedimentary rocks) due to the injection of a highly pressurised fluid. There is no existing reliable method to accurately measure fracture geometries during and after a HF treatment. Therefore, modelling and numerical simulations of the HF process play a vital role in the design and control of this technique.

During the early period of hydraulic fracturing two dimensional (2D) models with simplified geometries (e.g. KGD and PKN models) were proposed (Geertsma and de Klerk, 1969; Khristianovic and Zheltov, 1955; Nordgren, 1972; Perkins and Kern, 1961) to predict the shape and size of a hydraulic fracture based on properties of the rock and the fluid, the pumping parameters and the in situ stresses. Recent works along this line deal with the non-dimensional analysis of hydraulic fractures where different propagation regimes were discovered see Adachi and Detournay (2008), Bunger et al. (2005) among others. These analytical solutions prove to be extremely useful in the verification of numerical models.

There are a plethora of numerical methods developed to model hydraulic fractures ranging from classical methods such as finite element methods (FEMs), boundary element methods (BEMs), meshfree methods (Li et al., 2016; Nguyen et al., 2016), discrete element methods (DEMs) to recent techniques such as phase field modelling e.g. (Mikelić et al., 2015) and peridynamics (Oterkus et al., 2017). In the FEM community one can cite the cohesive interface elements (Boone and Ingraffea, 1990; Carrier and Granet, 2012; Chen et al., 2009; Guiducci et al., 2002; Papanastasiou, 1997; Segura and Carol, 2008a,b), the partition of unity methods (or XFEM) (de Borst et al., 2006; Gordeliy and Peirce, 2013; Mohammadnejad and Khoei, 2013; Remij et al., 2015a,b) and adaptive remeshing techniques (Fu et al., 2013; Schrefler et al., 2006). It is interestingly surprising to note that the aforementioned XFEM papers only solve problems with very simple crack geometries (most often a horizontal crack) even though XFEM is flexible to

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deal with complex crack patterns. To the best of the authors' knowledge only Fu et al. (2013) presented simulations with complex cracks using adaptive FEM and finite volume method for the fracturing fluid. The dominant fracture model in the FEM community is cohesive zone models (Barenblatt, 1962; Dugdale, 1960) as it removes the crack tip singularity of linear elastic fracture mechanics (LEFM). Benefits of FEM for HF simulations are its ability to deal with plastic deformation, poroelastic effects and heterogeneities.

Usually in hydraulic fracture simulators the rock formation is assumed to be homogeneous isotropic linear elastic (Adachi et al., 2007) and thus a BEM discretisation constitutes a very efficient method as only the boundary of the domain is discretised. Furthermore BEM is known to yield accurate results for LEFM which is the most commonly adopted fracture theory in hydraulic fracture codes. Typical BEM models for HF processes can be found in McClure and Horne (2014), Shen et al. (2013), Sousa et al. (1993), Wu and Olson (2014), Zhang et al. (2007), among others. As BEM was used the rock formation was restricted to (isotropic) linear elastic. However very complex crack patterns including the interaction of hydraulic fractures and existing natural ones can be handled efficiently e.g., McClure and Horne (2014). In the DEM community, the ITASCA FLAC and PFC2D codes have been widely adopted (Al-Busaidi et al., 2005; Bruno et al., 2001). The main advantage of DEM lies in the ability to handle complex crack patterns, to model fluid flow in both the pores and the fractures, and to deal with the fluid lag phenomenon as there are no ‘cracks' in DEM (Shimizu et al., 2011). And the disadvantages lie probably in the intensive, time-consuming calibration process to obtain the required material constants.

In this paper we revisit the cohesive interface elements which were presented in Guiducci et al. (2002). The referred paper was too short to provide practically useful details and did not deal with hydraulic fracture problems. The porous medium is modelled using the well known $u-p$ formulation that is based on Biot's poroelasticity theory (Biot, 1941) and the hydraulic fractures of which behaviour follows the cohesive zone theory (Barenblatt, 1962; Dugdale, 1960) are modelled with triple-noded interface elements. The mid-side nodes of these interface elements are used to model the longitudinal flow of the fracturing fluid. Our model employs a master-slave method to enforce the continuity of the pressure field across a fracture (if needed) whereas Lagrange multipliers were used (Carrier and Granet, 2012). The master-slave method was also utilised in Centeno Lobäo et al. (2010), for a horizontal crack, but a staggered solver was used. Herein, a monolithic solver where the displacements, the pore pressure and the fracturing fluid pressure are solved simultaneously. Besides, our implementation is generic as it applies to arbitrary crack patterns. Furthermore computational aspects such as (i) how to generate interface elements with fracturing fluid nodes, (ii) how the interfacial integrals are evaluated, (iii) performance of Gauss quadrature and Newton-Cotes rule and (iv) performance of quadratic interface elements in HF simulations, which are never reported in the open literature, are discussed. Also discussed are convergence property of the monolithic solver as well as execution time of HF simulations. We shall show that Newton-Cotes quadrature must be used for quadratic flow cohesive interface elements at least for the presented problems. Finally our simulations exhibit the so-called intermittent crack tip advancement as recently confirmed in Cao et al. (2017). In this paper, for simplicity, we restrict our investigation to (1) fully saturated media, (2) isothermal conditions, (3) small deformation, (4) no fluid-lag, and (5) two dimensions. All assumptions deem to be reasonable for HF processes except the fluid-lag one which deserves further comments. The zero fluid-lag assumption is valid if the confining stress is sufficiently large (Adachi and Detournay, 2008). The literature on modelling HF with fluid-lag is however scarce (Hunsweck et al., 2013; Lecampion and Detournay, 2007).

The remainder of the paper is organised as follows. The governing equations of the problem of fluid-driven fractures in porous media are given in Section 2 followed by Section 3 which is devoted to the corresponding weak forms. Finite element discretisation and discrete equations are presented in Section 4. Solution of these discrete equations is treated in Section 5, followed by implementation aspects presented in Section 6. Benchmark examples are provided in Section 7.

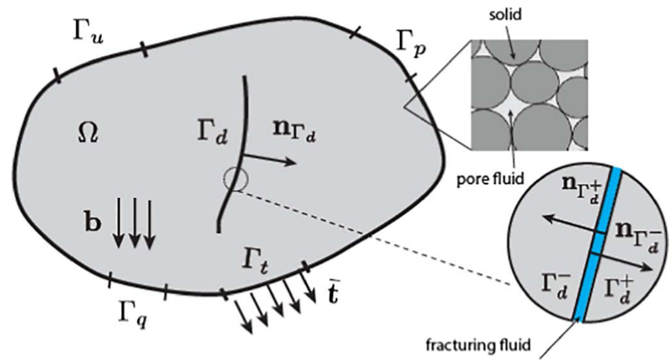


**Fig. 1.** A porous medium with a fluid-filled crack.

## 2. Fluid-driven fractures in porous media: governing equations

Hydraulic fracturing or more general fluid-driven fractures in porous media is a complex phenomenon which couple different physical processes. They include (i) flow of the fracturing fluid in the fractures, (ii) flow of the pore fluid in the porous medium, and (iii) deformation of the porous medium (including fracture initiation and propagation). In this paper, for simplicity, we confine to (1) fully saturated media, (2) isothermal condition, (3) small deformation, (4) no fluid-lag, and (5) two dimensions. We refer to Fig. 1 for a graphical illustration for the problem under investigation. The Biot's theory (Biot, 1941) is adopted to model the porous media and cohesive zone model (Barenblatt, 1962; Dugdale, 1960) is used to describe the fracture behaviour.

### 2.1. Deformation of porous media

The fully saturated porous medium is modelled as a two-phase system, where the pores of the solid skeleton are fully filled with a single phase fluid, refer to Fig. 1. The problem is formulated in terms of the solid displacements $\mathbf{u} \in \mathbb{R}^3$ and the pore fluid pressure $p_w \in \mathbb{R}$, that is the well known $u-p$ formulation is adopted. The following gradient and divergence operators used in subsequent developments are conveniently introduced $\nabla f = \frac{\partial f}{\partial x_i}\mathbf{e}_i$, and $\nabla \cdot \mathbf{u} = \frac{\partial u_i}{\partial x_i} = u_{i,i}$ where the indices $i$ run over the spatial directions, and they can be either $x$,$y$, and $z$, or 1,2, and 3. The Einstein's summation convention is applied by assuming summation over each pair of repeated indices.

In low-frequency ranges, the relative acceleration of the fluid phase with respect to the solid phase is negligible when compared with the acceleration of the solid phase. In this case, conservation of the linear momentum of the porous media domain Ω is described by the following equation

$$\nabla \cdot \boldsymbol{\sigma} + \rho \mathbf{b} - \rho \ddot{\mathbf{u}} = \mathbf{0} \tag{1}$$

where $\mathbf{b}$ is the body force, $\ddot{\mathbf{u}}$ denotes the acceleration vector of the solid; $\boldsymbol{\sigma}$ is the total stress tensor; ρ is the density of the porous medium which is given by

$$\rho = (1-n)\rho_s + n\rho_w \tag{2}$$

where $n$ is the porosity defined as the ratio of pore space to the total volume[1]; $\rho_s$ and $\rho_w$ are the intrinsic densities of the solid and the fluid, respectively.

[1] In most unconsolidated formation, porosity depends upon the grain size distribution; not on the absolute size of the grain itself.