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A stochastic computational method based on goal-oriented error estimation for heterogeneous geological materials

S.Sh. Ghorashi [d], T. Lahmer [d], A.S. Bagherzadeh [d], G. Zi [c], T. Rabczuk [a,b,d,*]

[a] Division of Computational Mechanics, Ton Duc Thang University, Ho Chi Minh City, Viet Nam
[b] Faculty of Civil Engineering, Ton Duc Thang University, Ho Chi Minh City, Viet Nam
[c] School of Civil, Environmental and Architectural Engineering, Korea University, Republic of Korea
[d] Institute of Structural Mechanics, Bauhaus-Universität Weimar, Weimar, Germany

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ABSTRACT

Computational modeling of geological materials is challenging. Firstly, they are heterogeneous with numerous uncertainties in the input parameters and secondly, the computational cost of modeling geological structures is time consuming due to the large and different length scales involved. In this article, we propose an efficient computational method for heterogeneous geological materials based on goal oriented error estimation and adaptive mesh refinement. Instead of estimating the error in a specific norm, the proposed novel error estimation approach which is called dual-weighted residual error estimation, approximates the error with respect to the quantity of interest. The dual-weighted residual error estimation is a dual-based scheme which requires an adjoint problem. The adjoint or dual problem is described by defining the quantity of interest in a functional form. Then by solving the primal and dual problems, errors in terms of the specified quantities are calculated. In many applications in engineering geology, the material is heterogeneous. In such cases, the material properties can be regarded as random fields. This variety of material properties leads to non-uniform distributions of the solution gradients, e.g. stresses. Therefore, it is vital to apply a reliable error estimation approach to be able to do efficiently the mesh-adaptivity procedure with regard to varying material parameters with pre-defined correlation lengths. Hence, the proposed error estimator is extended by accounting for a random field model to describe the material properties. Local estimated errors are exploited in order to accomplish the mesh adaptivity procedure. The goal-oriented mesh adaptivity controls the local errors in terms of the prescribed quantities. Both refinement and coarsening processes are applied to raise the efficiency. The performance of the proposed computational approach is demonstrated for several examples.

1. Introduction

Nowadays complicated engineering applications can be analyzed by numerical methods running on available computers. One of the most robust and reliable computational approaches is Finite Element Method (FEM). Although, many other numerical methods (Chen et al., 2012; Chau-Dinh et al., 2012; Ren et al., 2012; Vu-Bac et al., 2016; Quayum et al., 2015; Zhu et al., 2016) such as meshfree methods (Nguyen et al., 2008; Rabczuk and Belytschko, 2004; Ghorashi et al., 2011; Khazal et al., 2015; Amiri et al., 2014a, b; Rabczuk et al., 2010; Zhuang et al., 2012, 2014), isogeometric analysis method and its extensions (Hughes et al., 2005; Ghorashi et al., 2012a, b, 2015; Jia et al., 2014; Anitescu et al., 2015; Valizadeh et al., 2015; Ghasemi et al., 2015), efficient remeshing techniques (Areias et al., 2013a, b, 2014, 2015, 2016; Areias and Rabczuk, 2013; Nguyen-Xuan et al., 2013) or multiscale methods (Yang et al., 2015; Budarapu et al., 2014a, b; Talebi et al., 2013, 2014, 2015) have been introduced and successfully applied in different fields, FEM still plays an important role in computational mechanics field. For the special case of randomly varying material properties, the FEM can be enhanced to describe the true physical behavior of the material even more precisely by the use of random fields, a technique in particular of high importance in the field of reliability analysis.

In the FEM, mesh discretization highly affects the solution accuracy and obviously the computational effort. Therefore, it is of great importance to be able to minimize the computational cost while the expected solution accuracy is gained. Mesh adaptation is a profitable approach to achieve this goal. As a criterion for mesh configuration and updating, estimation of discretization error is required.

A good error estimator plays a very important role to implement an efficient refinement procedure in numerical methods. An error estimate should be performed to locate the situations of error distribution in the problem domain. The error estimation methods based on classical

* Corresponding author at: Division of Computational Mechanics, Ton Duc Thang University, Ho Chi Minh City, Viet Nam.
E-mail address: timon.rabczuk@tdt.edu.vn (T. Rabczuk).

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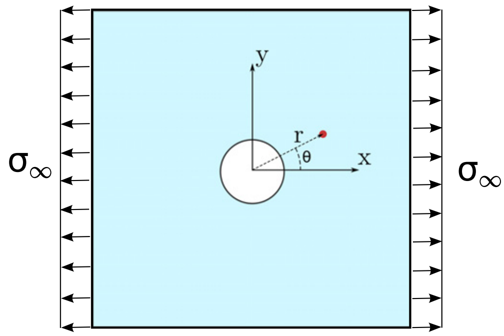

Fig. 1. The plate with hole subjected to far-field uni-directional tension.

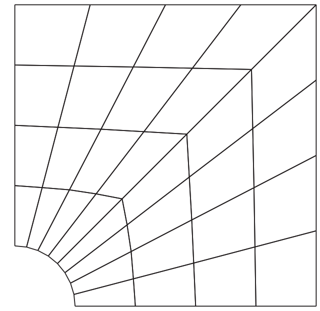

Fig. 3. Initial discretization of the plate with hole subjected to tension.

energy norm are categorized into two classes namely the residuals-based (Babuška and Rheinboldt, 1978; Amani et al., 2012, 2014) and the recovery-based (Zienkiewicz and Zhu, 1987) methods. In a residual-based error estimator, the residuals of a governing differential equation and its boundary conditions are considered as an error criteria while, the gradient of solutions is utilized in recovery-based methods (Zienkiewicz and Zhu, 1992a, b).

It is practically important to be able to estimate the error in the so-called quantity of interest (QoI) rather than the global energy norm. A new type of error estimation procedure called goal-oriented error estimation (GOEE) has been proposed to estimate the error with respect to the QoI (Becker and Rannacher, 1996, 1998; Bangerth and Rannacher, 2003; Prudhomme and Oden, 1999; Stein et al., 2007; Zaccardi et al., 2013; González-Estrada et al., 2014). It results in quantifying the effect of local errors on the accuracy of the solution with respect to the specific quantities. Therefore, this methodology is so beneficial for adaptivity schemes and quality assessment in engineering applications.

In this paper, the dual-weighted residual error estimation besides the conventional residual-based error estimation is applied in a three-dimensional elasticity problem. Local estimated errors are exploited in order to accomplish the local mesh adaptivity, refinement/coarsening, considering hanging nodes. The simulations are carried out by using the open source FEM library, deal.II (Bangerth et al., 2007). The goal-oriented mesh adaptivity controls the local errors in terms of the prescribed quantity. The convergence rates are plotted to illustrate the superiorities of the goal-oriented mesh adaptivity methodology over the traditional methods.

The rest of the paper is organized as follows: In Section 2, the goal-oriented error estimation based on dual-weighted residual is described. Section 3 describes one approach to generate random fields based on triangular decompositions of the covariance matrix. In Section 4, a numerical example considering both homogeneity and heterogeneity is investigated by applying different adaptivity strategies. Finally, some concluding remarks are outlined in Section 5.

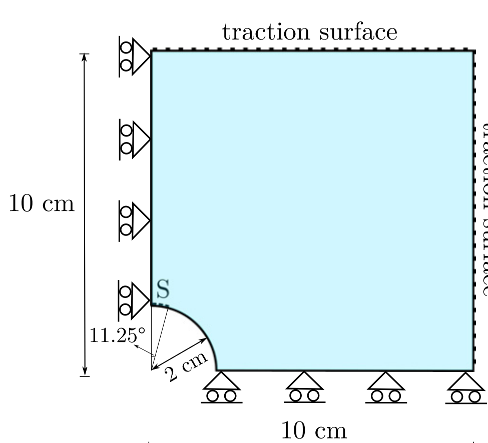

Fig. 2. Geometry and boundary conditions of the plate with hole subjected to tension.

2. Goal-oriented error estimation

In engineering applications the entire solution of the problem may not be interested, but rather some certain aspects of it. This is the main idea of applying the goal-oriented error estimation (GOEE). For example, in an elasticity problem one might want to know about values of the stress at certain points to predict whether maximal load values of joints are safe.

In GOEE procedure, solution of a dual/auxiliary/adjoint problem is also required.

2.1. Primal problem

Let us consider the following elastic equation:

$$-\partial_j \sigma_{ij} = f_i, \quad i,j,k,l = 1,2,3 \tag{1}$$

where the stress is defined as $\sigma_{ij} = c_{ijkl}\partial_k u_l$.

The displacement and traction boundary conditions can be written as

$$u_i = \overline{u}_i, \quad i = 1,2,3, \quad \text{on}\,\Gamma_d. \tag{2}$$

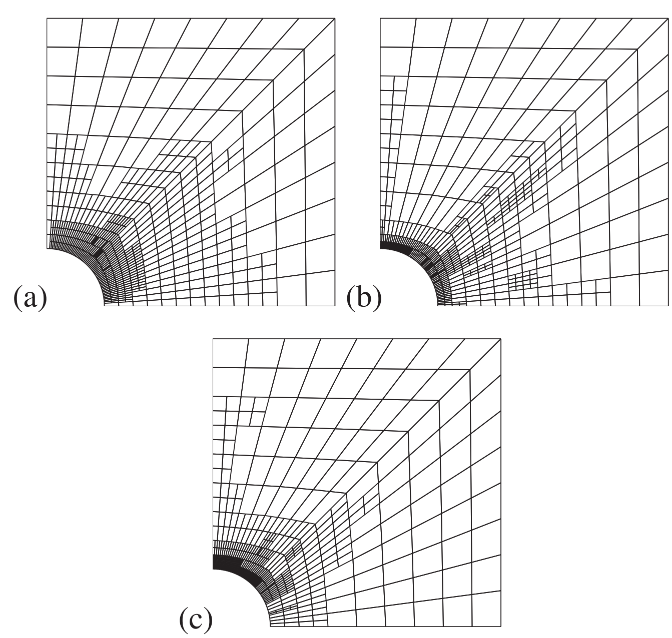

Fig. 4. Meshes at the 4-th adaptivity step of the plate with hole subjected to tension by applying: (a) Kelly refinement, (b) residual-based adaptivity and (c) DWR adaptivity.