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Review

Quantum hyperentanglement and its applications in quantum information processing

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A B S T R A C T

Hyperentanglement is a promising resource in quantum information processing with its high capacity character, defined as the entanglement in multiple degrees of freedom (DOFs) of a quantum system, such as polarization, spatial-mode, orbit-angular-momentum, time-bin and frequency DOFs of photons. Recently, hyperentanglement attracts much attention as all the multiple DOFs can be used to carry information in quantum information processing fully. In this review, we present an overview of the progress achieved so far in the field of hyperentanglement in photon systems and some of its important applications in quantum information processing, including hyperentanglement generation, complete hyperentangled-Bell-state analysis, hyperentanglement concentration, and hyperentanglement purification for high-capacity long-distance quantum communication. Also, a scheme for hyper-controlled-not gate is introduced for hyperparallel photonic quantum computation, which can perform two controlled-not gate operations on both the polarization and spatial-mode DOFs and depress the resources consumed and the photonic dissipation.

1. Introduction

Quantum information processing (QIP) has attracted considerable interest and attention of scientists in a variety of disciplines with its ability for improving the methods of dealing and transmitting information [1,2]. Entanglement is a distinctive feature of quantum physics [3], and it is very useful in QIP, including both quantum communication and quantum computation. Entangled photon systems are the natural resource for establishing quantum channel in long-distance quantum communication, especially in quantum repeaters [4] for some important tasks of communication, such as quantum key distribution [5–7], quantum secret sharing [8], and quantum secure direct communication [9–13]. In experiment, the entangled photon systems are usually prepared by the spontaneous parametric down-conversion (SPDC) process in nonlinear crystal [14–16]. In the conventional protocols for quantum information processing, the entanglement in one degree of freedom (DOF) of photon systems is selected in the SPDC process. In fact, there are more than one DOF in a quantum system, such as the polarization, spatial-mode, orbit-angular-momentum, frequency, and time-bin DOFs in a photon system.

Hyperentanglement, the simultaneous entanglement in multiple DOFs of a quantum system, has been studied extensively in recent years. It is a promising candidate for QIP with its high-capacity character. In experiment, hyperentanglement can be generated by the combination of the techniques used for creating entanglement in a single DOF [17]. With this method, many different types of hyperentangled states can be prepared [18–25], such as the polarization-spatial hyperentangled state [18], polarization-spatial-time-energy hyperentangled state [19], and so on. Hyperentanglement is a fascinating resource for quantum communication and quantum computation. On one hand, it can assist us to implement many important tasks in quantum communication with one DOF of photons, such as quantum dense coding with linear optics [26], the complete Bell-state analysis for the quantum states in the polarization DOF [27–31], the deterministic entanglement purification [31–34], and the efficient quantum repeater [35]. On the other hand, hyperentanglement can be used directly in some important applications in QIP. For example, it can improve the channel capacity of quantum communication and speedup quantum computation largely.

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In the applications of hyperentanglement, the complete hyperentangled-Bell-state analysis (HBSA) [36–45], hyper-teleportation of quantum state with more than one DOF [36], hyperentanglement swapping [37], hyperentanglement concentration [46–53], hyperentanglement purification [47,54–57], and universal entangling quantum gates for hyperparallel photonic quantum computation [58–62] are very useful and important. HBSA is the prerequisite for high-capacity quantum communication protocols with hyperentanglement and it is used to distinguish the hyperentangled states. Also, in the practical application of hyperentanglement in quantum communication, the hyperentangled photon systems are produced locally, which leads to the decoherence of the hyperentanglement when the photons are distributed over a channel with environment noise or stored in practical quantum devices. Quantum repeater is a necessary technique to overcome the influence on quantum communication from this decoherence [4]. In high-capacity quantum repeater with hyperentanglement, hyperentanglement concentration and hyperentanglement purification are two passive ways to recover the entanglement in nonlocal hyperentangled photon systems. They are not only useful but also absolutely necessary in long-distance high-capacity quantum communication with hyperentanglement as the self-error-rejecting qubit transmission scheme [63] do not work in depressing the influence of noise from both a long-distance channel and the storage devices for quantum states. Moreover, quantum repeaters for long-distance quantum communication require the entangled photons with higher fidelity (usually ~99%) beyond that from faithful qubit transmission schemes.

Different from conventional parallel quantum computation in which the states of quantum systems in one DOF or equivalent are used to encode information, hyperparallel photonic quantum computation performs universal quantum gate operations on two-photon or multi-photon systems by encoding all the quantum states of each photon in multiple DOFs (two or more DOFs) as information carriers [58–62]. With hyperparallel photonic quantum logic gates, the resource consumption can be reduced largely and the photonic dispassion noise can be depressed in quantum circuit [60]. Moreover, the multiple-photon hyperentangled state can be prepared and measured with less resource and less steps by using the hyperparallel photonic quantum logic gates, which may speedup the quantum algorithm [58,59].

In this review, we will overview the development of hyperentanglement and its applications in QIP in recent several years. We will first review the preparation of hyperentanglement, and then introduce the applications of the hyperentanglement in quantum communication, including hyper-teleportation of an unknown quantum state in more than two DOFs and hyperentanglement swapping. We also highlight how to improve the entanglement of nonlocal hyperentangled photon systems with hyperentanglement concentration and hyperentanglement purification. At last, the principle of a polarization-spatial hyper-controlled-not (hyper-CNOT) gate is described for hyperparallel quantum computing.

2. Preparation of hyperentanglement

Hyperentangled states offer significant advantages in QIP due to the presence of quantum correlations in multiple DOFs. In this section, we will introduce the preparation of hyperentangled states of photon systems. In the first part, we overview the preparation of entangled photon pairs with the SPDC process in nonlinear crystals. In the second part, we overview the preparation of hyperentangled photon systems with the combination of the techniques used for creating entanglement in single DOF.

2.1. Preparation of entanglement in single DOF

Generally speaking, the most extensive method used to generate an entangled state is the SPDC process in a nonlinear crystal. When a pump laser beam p shines a nonlinear birefringent crystal, the idler photon i and the signal photon s are generated probabilistically from the crystal. The maximal probability can be achieved by satisfying two matching conditions. One is the phase-matching:

$$\vec{k}_p = \vec{k}_s + \vec{k}_i, \tag{1}$$

and the other is energy-matching:

$$\omega_p = \omega_s + \omega_i. \tag{2}$$

Here $\vec{k}$ represents the wave vector and ω denotes the frequency. Usually, there are two common kinds of phase-matching adopted in experiment, depending on the extraordinary (e) and the ordinary (o) polarizations of the pump photon and the two SPDC photons. The type-I phase-matching is $e \rightarrow o + o$ and the type-II phase-matching is $e \rightarrow e + o$.

In the type-I phase-matching, two SPDC photons are both ordinary and have the same polarizations. To generate an entangled state, two crystals with orthogonal optical axes can be used [15]. The principle is shown in Fig. 1. To satisfy the phase-matching condition, two correlated photons are emitted over opposite directions of the cone surface. By selecting one pair of the correlated wavevector modes, the polarization entangled states $|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|HH\rangle \pm |VV\rangle)$ can be prepared. Here H and V represent the horizontal and vertical polarization states of a photon, respectively. An alternative way to prepare an entangled state with type-I phase-matching is using a single crystal and a double passage of the laser beam after reflection on a mirror [16].

In the type-II phase-matching, the two degenerate photons are emitted over two different mutually crossing emission cones [14]. The emission directions of the signal and idler photons are symmetrically oriented with respect to the propagation direction of the pump photon. The two entangled photons are generated along the direction of the intersection of the two cones. Since the ordinary and extraordinary photons have orthogonal polarization states, the polarization entangled states $|\Psi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|HV\rangle \pm |VH\rangle)$ are prepared with type-II phase-matching. If the two cones only intersect at one point, it is called the collinear SPDC process and orthogonally polarized photons are indistinguishable at exactly this point. The type-II collinear down-conversion is more commonly used in experiment, as it offers a trivial way to deterministically separate the photon pair by their polarization and to work with each photon separately. For the non-collinear type-II SPDC process which is shown in Fig. 2, the two emission cones have two intersection directions, which can be made indistinguishable

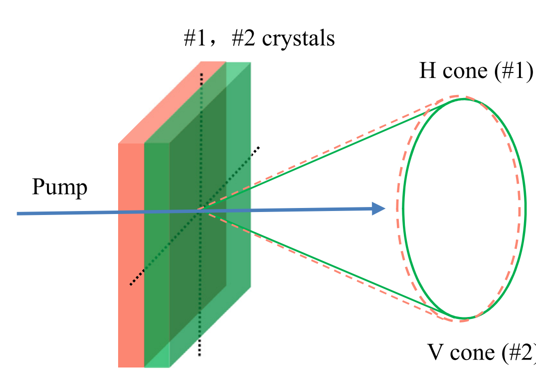

Fig. 1. (Color online) Type-I polarization entanglement sources [15].