Science Bulletin 62 (2017) 869–874

Contents lists available at ScienceDirect

Science Bulletin

journal homepage: www.elsevier.com/locate/scib

Article

Time reversal of a discrete system coupled to a continuum based on non-Hermitian flip

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A R T I C L E I N F O

Article history:
Received 30 March 2017
Received in revised form 2 May 2017
Accepted 8 May 2017
Available online 15 May 2017

Keywords:
Non-Hermitian dynamics
Time reversal
Loschmidt echo

A B S T R A C T

Time reversal in quantum or classical systems described by an Hermitian Hamiltonian is a physically allowed process, which requires in principle inverting the sign of the Hamiltonian. Here we consider the problem of time reversal of a subsystem of discrete states coupled to an external environment characterized by a continuum of states, into which they generally decay. It is shown that, by flipping the discrete-continuum coupling from an Hermitian to a non-Hermitian interaction, thus resulting in a non unitary dynamics, time reversal of the subsystem of discrete states can be achieved, while the continuum of states is not reversed. Exact time reversal requires frequency degeneracy of the discrete states, or large frequency mismatch among the discrete states as compared to the strength of indirect coupling mediated by the continuum. Interestingly, periodic and frequent switch of the discrete-continuum coupling results in a frozen dynamics of the subsystem of discrete states.

1. Introduction

The physics of open quantum and classical systems is of great relevance in different areas of science [1–13] ranging from atomic and molecular physics [5,7,8] to nuclear physics [1,3,4,6], dissipative systems driven out of equilibrium and quantum thermodynamics [14,15], quantum computing [7,16,17], transport in nanoscale and mesoscopic solid state devices [9,10,18–24], optics and photonics [25–29], and transport in biological structures [30–32]. More recently, non-Hermitian models have been also introduced in *ab initio* theories to provide complex extensions of quantum mechanics [33–39]. Studies of open quantum systems date back to some pioneering theoretical works, notably of Gamow, Weisskopf-Wigner and Feshbach on nuclear α-decay [1], spontaneous emission [2], and nuclear reactions [3], are nowadays of continuous interest in a wide range of different areas of physics. In a typical situation, a discrete system described by N states is coupled to an external environment characterized by a continuum of states, into which they generally decay. An important example if provided, for instance, by electronic or photonic transport in mesoscopic or macroscopic cavities with attached wave guides. A simple approach to describe open systems is to eliminate the continuum degrees of freedom via Feshbach projection technique or similar methods [2–6,8,9], leading to an effective non-Hermitian Hamiltonian description for the finite-dimensional N subspace of discrete states. Such an approach turns out to be very effective in predicting important interference effects between discrete states and the continuum, such as Fano resonances, the existence of bound states in the continuum, external mixing of states, nonadiabatic dynamical transitions near exceptional points, nonlinear effects, etc. (see, for example, [10–12] and references therein for a comprehensive overview). A major question is whether one can time-reverse the dynamics in the *reduced* N-dimensional subspace of discrete states. For example, is it possible to reverse the decay process of an initial excitation of the N discrete states into the continuum of states, so as to retrieve the initial excitation in the subspace of discrete states? The problem of time reversal in systems with many degrees of freedom is a longstanding one in physics and dates back to the famous controversy between Loschmidt and Boltzmann about the second law of thermodynamics and the arrow of time, i.e. how macroscopic irreversibility can arise even if dynamical equations of motion are time reversible. In principle, performing time reversal in an Hamiltonian system would "simply" require inverting the sign of the Hamiltonian. Quantum or classical reversibility in presence of various perturbations has been actively studied in recent years and is now described through the so-called Loschmidt echo [40–42]. In particular, time reversal dynamics has been experimentally demonstrated in a wide variety of quantum and classical systems, including spin systems [43], matter waves [44,45], acoustic [46], electromagnetic [47] and water waves [48]. Reversing the sign of the Hamiltonian, however, is not a simple task, especially when infinite degrees of freedom like those of a

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http://dx.doi.org/10.1016/j.scib.2017.05.012

continuum of states are involved and perturbations can rapidly deteriorate the Loschmidt echo.

In this work, it is suggested that reversing the dynamics of a discrete system coupled to a continuum can be achieved by flipping the discrete-continuum interaction from an Hermitian to a non-Hermitian coupling. Physical models and practical implementations of non-Hermitian couplings have been considered in some recent works [49–55], especially in the context of tight-binding lattice models. A non-Hermitian coupling does not generally conserve the probability of the discrete-continuum system, since another hidden reservoir is involved in the dynamics from which energy and/or particles can be fed or extracted. As far as time reversal dynamics is concerned, flipping the coupling of the discrete states with the continuum from an Hermitian to a non-Hermitian one can reverse, under certain conditions, the dynamics of the discrete states, but not the one of the continuum which can take energy from the hidden reservoir. Remarkably, periodic flipping of the coupling over a short time scale can result in a frozen dynamics of the N discrete states, but not of the continuum of states. As an example, we discuss time reversal and frozen dynamics based on non-Hermitian flip in a multilevel Fano-Anderson model describing the coupling of N sites to a tight-binding continuum.

2. Time reversal based on non-Hermitian flip

2.1. Basic model and effective Hamiltonian

Let us consider a discrete system of N states $|a_n\rangle$ with frequencies ω_n $(n = 1, 2, \ldots, N)$ which are coupled to an external environment characterized by a continuum of states $|\omega\rangle$ with frequency ω. The Hamiltonian of the full system is described by the multilevel Fano-Anderson (or Friedrichs-Lee) Hamiltonian [56]

$$\hat{H} = \sum_n \omega_n |a_n\rangle\langle a_n| + \int d\omega\omega|\omega\rangle\langle\omega| + f(t)\sum_n \{g_n(\omega)|a_n\rangle\langle\omega| + g_n^*(\omega)|\omega\rangle\langle a_n|\}, \tag{1}$$

where $g_n(\omega)$ is the spectral amplitude that describes the coupling of the discrete state $|a_n\rangle$ with the continuum $|\omega\rangle$ and $f(t)$ defines the coupling strength, which is generally assumed to be time dependent. The Hamiltonian $\hat{H}$ is Hermitian provided that $f(t)$ is real. In this case the dynamics is unitary and the norm (probability) is conserved. However, rather generally we allow $f(t)$ to become complex, corresponding to a non-Hermitian discrete-continuum coupling and a non-unitary dynamics. It should be noted that a non-Hermitian coupling requires a hidden reservoir into which the discrete-continuum states can transfer particles/energy excitation, thus explaining the non-unitary dynamics and breakdown of population conservation. Non-Hermitian coupling by means of a complex (imaginary) function $f(t)$ is introduced here at a phenomenological level, i.e. without a detailed description of the “hidden” reservoir which is eliminated from the dynamics and that makes sense of an effective non-Hermitian discrete-continuum interaction. Indeed, the problem of time reversal of discrete states discussed in the following and based on the weak-coupling approximation does not require a detailed microscopic description of the non-Hermitian interaction. For example, photonic transport in coupled waveguides or optical resonators is often described in the framework of the Fano-Anderson model [57,58]. Here the hidden reservoir is provided by the pumped atomic medium that provides instantaneous spatial optical gain and loss regions in the system and that makes the dynamics effectively non-Hermitian. In particular, an imaginary (non-Hermitian) coupling of wave guides can be implemented using the methods discussed in Refs. [50,54,59]. Similarly, in non-Hermitian models of mesoscopic quantum transport imaginary potentials that act as source and sink for carriers are phenomenologically introduced, without the need for a full description of the entire reservoir dynamics [60,61].

If the state vector of the system $|\psi(t)\rangle$ is expanded on the discrete-continuum basis as

$$|\psi(t)\rangle = \sum_n c_n(t)|a_n\rangle + \int d\omega c(\omega, t)|\omega\rangle, \tag{2}$$

from the Schrödinger equation with $\hbar = 1$ one obtains the following evolution equations for the amplitudes $c_n(t)$ and $c(\omega, t)$

$$i\frac{dc_n}{dt} = \omega_n c_n(t) + f(t)\int d\omega g_n(\omega)c(\omega, t), \tag{3}$$

$$i\frac{dc}{dt} = \omega c(\omega, t) + f(t)\sum_n g_n^*(\omega)c_n(t). \tag{4}$$

Let us assume that at initial time the continuum of states is empty, i.e. $c(\omega, 0) = 0$. The formal integration of Eq. (4) yields

$$c(\omega, t) = -i\sum_n g_n^*(\omega)\int_0^t d\xi f(\xi)c_n(\xi)\exp[-i\omega(t-\xi)]. \tag{5}$$

After setting $c_n(t) = a_n(t)\exp(-i\omega_n t)$, substitution of Eq. (5) into (3) yields the following set of integro-differential equations for the amplitudes $a_n(t)$

$$\frac{da_n}{dt} = -f(t)\sum_m \int_0^t d\xi f(\xi)a_m(\xi)\Phi_{n,m}(t-\xi)\exp(i\omega_n t - i\omega_m \xi), \tag{6}$$

where we have introduced the memory functions

$$\Phi_{n,m}(\tau) \equiv \int d\omega g_n(\omega)g_m^*(\omega)\exp(-i\omega\tau). \tag{7}$$

The characteristic decay time τ_{mem} of $\Phi_{n,m}(\tau)$ defines the memory time of the response function. Following a standard procedure, in the Weisskopf-Wigner (markovian) approximation, corresponding to a weak coupling $g_n(\omega) \to 0$, a broad and unstructured continuum, and a small change of $f(t)$ over the memory time τ_{mem}, the integro-differential Eqs. (6) can be replaced by the following set of differential equations

$$\frac{da_n}{dt} = -f^2(t)\sum_m \Delta_{n,m}a_m(t)\exp[i(\omega_n - \omega_m)t], \tag{8}$$

where we have set

$$\begin{aligned}\Delta_{n,m} &= \int_0^\infty d\tau\Phi_{n,m}(\tau)\exp(i\omega_m\tau) \\ &= \pi g_n(\omega_m)g_m^*(\omega_m) - iP\int d\omega\frac{g_n(\omega)g_m^*(\omega)}{\omega - \omega_m}.\end{aligned} \tag{9}$$

Eq. (8) provides an effective non-Hermitian description of the dynamics in the subspace of the N discrete states $|a_n\rangle$, in which the degrees of freedom of the continuum of states $|\omega\rangle$ have been eliminated in the markovian approximation. For an Hermitian and time-independent Hamiltonian $\hat{H}$, i.e. for $f(t) = 1$, the real and imaginary parts of the eigenvalues λ_n of the effective coupling matrix $\mathcal{D} = \{\Delta_{n,m}\}$ determine the decay rates and frequency shifts of the resonance modes. Such an effective non-Hermitian description is able to capture some important effects such as the existence of bound (trapping) states inside the continuum and Fano resonances arising from the interference of the discrete states via the continuum. It should be noticed that the above analysis is based on the rather standard markovian approximation, i.e. the solutions are obtained by assuming a weak coupling between the discrete states and the continuum. This means that the validity of the results discussed in the next sections, i.e. time reversal and frozen dynamics by non-Hermitian flip, is strictly speaking restricted to the weak