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Article

Negative exponential behavior of image mutual information for pseudo-thermal light ghost imaging: observation, modeling, and verification

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ABSTRACT

When using the image mutual information to assess the quality of reconstructed image in pseudo-thermal light ghost imaging, a negative exponential behavior with respect to the measurement number is observed. Based on information theory and a few simple and verifiable assumptions, semi-quantitative model of image mutual information under varying measurement numbers is established. It is the Gaussian characteristics of the bucket detector output probability distribution that leads to this negative exponential behavior. Designed experiments verify the model.

1. Introduction

Image quality assessment is known to be difficult so far [1]. Besides traditional image quality measures based on error estimation, assessments of different types are introduced, first by the signal processing community (cf. [2] for a review). Among others, the mutual information (MI), representing the amount of information shared by two random variables in information theory [3], was introduced to account the similarity between images [4,5], and has been successfully applied in different circumstances to assess image quality [6,7].

Being different from the usual "single snapshot" imaging process, ghost imaging (GI) is built on a large number of consecutive measurements on two quantities: light intensity registered by a "bucket" detector with no spatial resolution, and a spatial profile that never reaches the object—either an "idler" reference light field [8], a modulation pattern [9], or the calculated diffraction profile of that field [10]. As a consecutive process, modeling the performance under varying measurement numbers is of great significance to GI. One would naturally expect the image quality to improve with increasing measurement number n, and converge when n is quite large, suggesting an upper limit of image quality when $n \to \infty$. Unfortunately, previous studies of image quality focus on the influence of either the noise level [11–13] or relative spatial/temporal scale [14], and no quantitative analysis concerning measurement number has been published according to our knowledge, except for a few qualitative observations [9,15] and an untight lower bound [16].

GI has been applied to various scenarios, ranging from entangled photon pairs [8], pseudo-thermal light [17], thermal light from hollow-cathode lamp [18], sunlight [19], all the way up to X-ray [20,21]. Although different types of GI image quality assessments have been studied and compared, e.g., mean square error (MSE) [9], signal-to-noise ratio (SNR) [14], and contrast-to-noise ratio (CNR) [22], as Ref. [23] pointed out, these are all "posterior" assessments depending on specified image content, i.e., one has to get full knowledge of the "ideal" pattern beforehand, thus are inappropriate for design and optimization of a general imaging system. Ref. [23] pioneers a promising subject of prior GI image quality assessment by introducing MI between a random object and its GI image. However, they simply assumed that MI grew linearly with the measurement number n, which had not been examined, and appears to be not true according to our experiment.

In this contribution, we use image mutual information (IMI) between the object O and the reconstructed image Y to assess image quality of pseudo-thermal light GI. Semi-quantitative fitting shows that IMI $I(O;Y)$ is a negative exponential function of the measurement number n. An information-theory-based model explains this behavior. All the assumptions are validated. Designed

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further experiments demonstrate highly agreement with the predictions of the model.

2. Methods and observation

2.1. Experiment setup

A conventional GI setup is implemented as in Fig. 1. Output of a 532 nm laser passes through a rotating ground glass (R.G.G., Edmund 100 mm diameter 220 grit ground glass diffuser), turning it into pseudo-thermal light [24], whose intensity fluctuates randomly both in the space and time domain. This pseudo-thermal light is then split into two arms by the beam splitter (BS). The signal arm penetrates a transmissive object mask, followed by a focus lens, to be registered as a whole into a temporal intensity sequence $B(t)$ by a bucket detector which has no spatial resolution. The spatial profile of the reference arm, $R(x;t)$, which never reaches the object, is recorded by a commercial CMOS camera (Thorlabs DCC3240C) synchronically with the bucket detector. The second order fluctuation correlation (2nd FC) [25] between corresponding $B(t)$ and $R(x;t)$ yields the reconstructed image $Y(x)$,

$$Y(x) \propto \frac{\left\langle \left[R(x;t) - \langle R(x;t)\rangle_t\right] \times \left[B(t) - \langle B(t)\rangle_t\right]\right\rangle_t}{\langle R(x;t)\rangle_t \langle B(t)\rangle_t}, \tag{1}$$

where $\langle\cdot\rangle_t$ denotes average over all the measurements.

2.2. Image mutual information

Mutual information In information theory, MI between two random variables A and B is defined as

$$I(A;B) = H(A) - H(A|B), \tag{2}$$

where $H(A) = -\sum_a p_A(a)\log_2 p_A(a)$ is the Shannon entropy of A with probability distribution function (PDF) $p_A(a)$, denoting the amount of information one reveals when gets full knowledge of $p_A(a)$, and $H(A|B)$ is the conditional entropy of A given B, representing the amount of the remain unknown information of A even when the probability distribution of B is totaly determined,

$$H(A|B) = -\sum_a \sum_b p_{A,B}(a,b)\log_2 p_{A|B}(a|b), \tag{3}$$

where $p_{A,B}(a,b)$ is the joint probability of $A=a$ and $B=b$, and $p_{A|B}(a|b)$ is the conditional probability of $A=a$ given $B=b$. Eq. (2) shows that $I(A;B)$ denotes the amount of information shared by two partite A and B, thus can be a measure of how similar the two variables are, since identical variables have the largest MI, while totally independent ones have the smallest.

Image mutual information When IMI is applied, image $A(x)$ of N pixels is treated as a one-dimensional random variable A of length N. IMI between two images $A(x)$ and $B(x)$ is defined as MI between two random variables A and B, which denotes the similarity between the two images. If $A(x)$ and $B(x)$ are set to be the object and image of an imaging system, respectively, IMI can assess the image quality, since the goal of imaging is to accomplish a duplicate as similar to the object as possible. In fact, by maximizing IMI, imaging distortion and relative displacement can be corrected profoundly—known as the image registration technique (cf. [6] for a review). Here we want to note that, in order to reduce the influence made by image distortion or relative displacement on the image quality assessment, the image and object should be aligned at first when uses IMI to assess image quality. What is more, unlike other assessments, e.g., mean square error (MSE), IMI is indifferent to any change of prior knowledge within the area of interest (AOI), i.e., specified object spatial profile $Y(x)$. This unique property, on one hand, emphasizes the importance of image alignment, and suggests the potential to develop a content-free image quality assessment on the other, which is a general measure of image quality regardless of what pattern it has for a specified image.

2.3. Observation of negative exponential behavior

Reconstructed image $Y(x)$ is recorded against different measurement numbers. The AOI contains 120×120 pixels. To ensure alignment, the image that is the most over-sampled $(n = 50{,}000), Y_\infty(x)$, serves as an almost-identical approximation of the object $O(x)$, assuming that after so many measurements, the image has been a stable, nearly perfect duplicate to the object. IMI between $O(x)$ and $Y(x), I(O;Y)$, is calculated under varying n to assess the image quality—the higher $I(O;Y)$ is, the better quality image one gets. For our system, the image quantization bit length when calculates IMI is set to be 9, according to the first part of the Electronic Appendix (Online). The result is shown in Fig. 2. Curve fitting with both linear and nonlinear regression shows that the negative exponential function fits the experiment result best, i.e.,

$$I(O;Y) = C_1 - C_2 \exp\left(-\frac{n}{C_3}\right), \tag{4}$$

where fitting parameter C_1 denotes the upper limit of $I(O;Y)$ when $n \to \infty$, and parameter C_3 represents the converge speed, i.e., C_3 measurements are required to reduce the uncertainty between image and object to the $1/e$ of its initial value. The larger C_3 is, the more measurements one needs to achieve the same level of image quality.

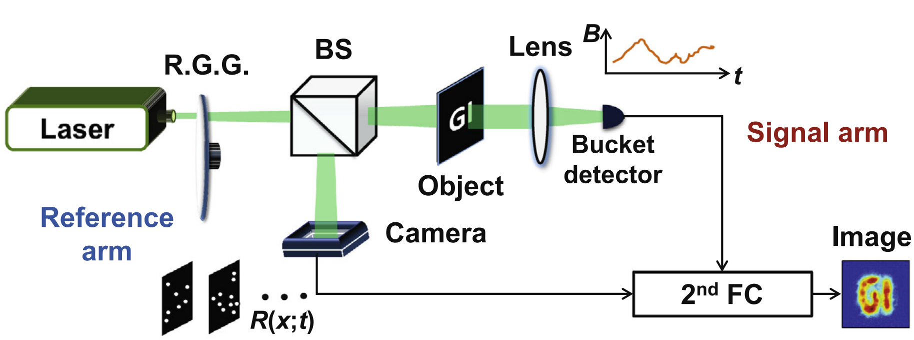

Fig. 1. (Color online) Experiment setup. R.G.G. = rotating ground glass. BS = beam splitter.

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