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Experimental verification of genuine multipartite entanglement without shared reference frames

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Abstract Quantum entanglement is an essential resource for quantum information processing, either for quantum communication or for quantum computation. The multipartite case of entanglement, especially the so called genuine multipartite entanglement, has significant importance for multipartite quantum information protocols. Thus, it is a natural requirement to experimentally verify multipartite quantum entanglement when performing many quantum information tasks. However, this is often technically challenging due to the difficulty of building a shared reference frame among all involved parties, especially when these parties are distant from each other. In this paper, we experimentally verify the genuine tripartite entanglement of a three-photon Greenberger-Horne-Zeilinger state without shared reference frames. A high probability 0.79 of successfully verifying the genuine tripartite entanglement is achieved when no reference frame is shared. In the case of sharing only one common axis, an even higher success probability of 0.91 is achieved.

Keywords Genuine multipartite entanglement · Reference-frame independent · Mermin-Klyshko inequality

1 Introduction

Quantum entanglement, which arises from applying the superposition principle on compound systems, is one of the most important resources in quantum communication and quantum computation. With the help of entanglement, many classically impossible task can be successfully performed, such as teleportation [1, 2], quantum superdense coding [3–5], etc.

Bipartite entanglement has been studied extensively and is well understood in many aspects, both in theory [6–10] and experiment [11–13]. However, the case of multipartite entanglement is far less than being well understood due to its complexity and much richer structures. In fact, many definitions for bipartite entanglement can not be directly generalized to the multipartite case and new concepts need to be introduced, e.g., genuine multipartite entanglement (GME), partial separability, etc. GME means the negation of all possible mixtures of bipartitions. An important difference between multipartite and bipartite entanglements is that there are different classes of entanglement for multipartite case. For example, two kinds of entanglement, i.e., the Greenberger-Horne-Zeilinger (GHZ) type and the W-state type [14–16], can be classified for three-qubit pure states [17], and four-qubit states can be entangled in nine different ways [18]. For more parties, there are no clear classifications up to now. Considering of such problems as the classification of multipartite entanglement, one further question of importance is to ask whether the entanglement

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property persists across all the subsystems, i.e., whether the entanglement is “truly multipartite” [19].

There have been various methods for verifying multipartite entanglement, such as Bell-like inequality [20, 21], quantum witness [22], entropies criteria [23], estimating the concurrence [24], local uncertainty realtions [25]. However, verifying entanglement of different parties with these methods normally requires shared reference frames among all the parties, which is usually a hard task if the entangled state is distributed distantly or there is no well defined reference frames [26, 27]. Recently, it is shown [28, 29] that in the case of sharing no reference frames it is still possible to verify multipartite entanglement property with a high success probability, even with certainty, using the Mermin-Klyshko (MK) inequality [30] as a witnesses, by applying certain measurement schemes or the form of correlation tensor norms [31].

Here, we experimentally verify the genuine tripartite entanglement property of a three-photon GHZ state without shared reference frames, by employing a tetrahedral measurement scheme introduced in Ref. [29], using the MK inequality as the witness. The experiment results give out a high probability 0.76 of successfully verifying the GHZ state’s genuine multipartite entanglement. And we also study another frequently encountered practical situation of sharing only one common axis. By applying a two-strategy measurement scheme, our experiment results give out a very high success probability 0.91. Furthermore, we also numerically analyze the affections of various noises on the success probabilities in such schemes.

2 Theoretical schemes

For simplicity, here we discuss the tripartite systems first, while the n-partite case can be extended directly. The state of tripartite systems can be classified through entanglement into three classes. Suppose that the state is described by a density matrix ρ, which in general can be decomposed as a mixture of pure states $\rho = \sum p_i|\psi_i\rangle\langle\psi_i|$, where $\sum p_i = 1$. Then, if we check through all possible such decompositions, there will be three different cases [32]: (1) ρ is not entangled at all. (2) ρ has bipartite entanglement. (3) ρ has true tripartite entanglement.

However, it is in fact impossible to cover all possible decompositions to tell which class ρ belongs to. Alternately, we can adopt a sufficient criterion to detect it, which is developed by Mermin [20]. Their criterion comes out as an inequality, the MK inequality, which denotes the entanglement classification as GME(m), $1 \le m \le n$ for n-parties. Here, the parameter m means that the state ρ contains genuine m-partite entanglement at most. The MK inequality can be constructed by writing the MK polynomial first [32]. In the n-partite case, the MK polynomial is given by $(P_{\mathrm{MK}})_n := \frac{1}{2}(P_{\mathrm{MK}})_{n-1}(a_n + a'_n) + \frac{1}{2}(P_{\mathrm{MK}})'_{n-1}(a_n - a'_n)$, where a_i and a'_i are some desired measurements, and $(P_{\mathrm{MK}})'_k$ is founded by exchanging all a_i and a'_i in $(P_{\mathrm{MK}})_k$. The basic polynomial $(P_{\mathrm{MK}})_2$ is given as $(P_{\mathrm{MK}})_2 = \frac{1}{2}(a_1a_2 + a'_1a_2 + a_1a'_2 - a'_1a'_2)$. Then, each product term in the polynomial, e.g., $a'_1a_2a_3$, is replaced by its expectation value, such as $E(a'_1a_2a_3)$. The absolute value of this new expression derived from the MK polynomial is denoted as $(\mathcal{I}_{\mathrm{MK}})_n$. And the MK inequality writes that $(\mathcal{I}_{\mathrm{MK}})_n \le 1$, means that there is no entanglement in the state. Now, for the n-partite state, if $(\mathcal{I}_{\mathrm{MK}})_n \le 2^{\frac{m-1}{2}}$ (where $1 \le m \le n$) [33], it implies that the state’s largest entangled subspace contains no more than m parties and any violation of this bound means that there is genuine $(m+1)$-partite entanglement in ρ, denoted as GME($m+1$). For the tripartite case, the violation of $(\mathcal{I}_{\mathrm{MK}})_3 > \sqrt{2}$ indicates the existence of genuine tripartite entanglement.

Obviously, measuring the MK inequality correctly relies on sharing a common reference frame among all parties [26, 27]. In the case of no shared reference frames, successfully verifying the GME(n) property by a given multipartite state requires some special methods [29, 30, 34]. In Ref. [29], the authors proposed a tetrahedral type bases, which contained 4 measurement directions and each corresponded to a vertex of the inscribed tetrahedron of the Bloch sphere. In their scheme, a n-qubit GHZ state was initially shared among the n parties. And the effect of sharing no reference frames was simulated by performing a random local rotation R_k on the kth subsystem before measuring them. Here, R_k is given as $R_k = \cos\frac{\theta_k}{2}I\!I - \mathrm{i}\sin\frac{\theta_k}{2}\left(n_k^1\sigma_1 + n_k^2\sigma_2 + n_k^3\sigma_3\right)$, where θ_k and n_k^j are real, $\sum_j n_k^j = 1$, and σ_k $(k = 1, 2, 3)$ are the Pauli operators. Then, each observer performs the four measurements of the tetrahedral vertices on his own qubit. Note that the random local rotation R_k is the same for all four measurements, i.e., R_k is fixed in each trial to evaluate the MK inequality value and randomized among different trials. So, for the n-partite system, there are totally 4^n joint measurement settings. After all these joint measurements have finished, we can obtain $(4 \times 3)^n$ MK inequality values, and each value is evaluated by picking up two out of the four measurement bases for each observer and substituting the corresponding joint measurement results for the expectation values in the MK inequality. At last, we need to check through all the $(4 \times 3)^n$ MK inequality values and pick out the maximal one to see whether it violates the inequality. If yes, this trial will be a successful verification. After a large number of trials, an estimation of the success probability of verification can be made by counting the number of successful trials over the total number of trials. This tetrahedral-bases scheme results in a higher success probability of verification than any other previous schemes. Concerning the