NUMERICAL TECHNIQUES FOR SPEEDING UP THE CALCULATION OF THE LIFE EXTENSION BROUGHT ABOUT BY REMOVING A PROLONGED RADIATION EXPOSURE

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Abstract: The judgement- or J-value, which enables the worth of any health or safety scheme to be measured on a common, objective scale, may be applied to a scheme to reduce or eliminate a prolonged radiation exposure provided the life extension achieved can be calculated. The calculation is necessarily complex because of the long and stochastic incubation periods associated with radiation-induced cancers. However, numerical techniques are presented here that speed up the calculation of the improved life expectancy by a factor of about one hundred. The J-value assessment of new safety systems on nuclear plant is thus made much quicker and easier.

Keywords: health; safety; nuclear; radiation; prolonged release; life expectancy; risk; J-value.

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INTRODUCTION

The J-value system can be used to determine acceptable health and safety expenditure across different sectors of the economy (Thomas *et al.*, 2006a, b). It works by taking into account the life extension brought about by the safety measure, as well as base-line values for life expectancy, average income and work-life balance. A J-value of below one indicates that the money spent is commensurate with the lives it saves or that spending more money may be beneficial. A J-value of over one indicates that the spending is out of proportion with the lives it can save, and that the money being spent could be better redirected to other areas.

The extension in life expectancy brought about by removing a prolonged radiation exposure must be calculated before one can estimate the J-values associated either with nuclear regulatory recommendations (Thomas *et al.*, 2006a) or with specific safety systems in the nuclear power industry (Thomas *et al.*, 2006b). The calculation of life extension is based the premise that any deaths caused by nuclear radiation will be delayed by at least a decade, but can then occur randomly over the next 30 years, with a uniform probability distribution within this interval (Marshall *et al.*, 1982; Thomas *et al.*, 2006c). It is assumed that there is a linear relationship between dose and the probability of harm, with the total risk coefficients taken from the 1990 recommendations of the International Commission on Radiation Protection (ICRP, 1990). The conservative basis of the ICRP figures means that the calculated change in life expectancy is likely to be somewhat high, which adds to the conservatism of nuclear J-values.

Methods for calculating the increase in life expectancy were given in detail in Thomas *et al.* (2006c). Improvements to the ease and speed of calculation were described in Thomas *et al.* (2007), based on analytic methods to convert a double integral needing numerical evaluation to a single such integral. This companion paper presents an alternative method of generating a similar, hundred-fold reduction in the size of the calculation, this time based on numerical techniques that approximate the notional, continuous exposure by a series of point exposures. The extent of the approximation will be quantified theoretically and by comparison with results produced by analytical methods (Thomas *et al.*, 2007).

Algebraic symbols are explained where they arise, but the definitions are included in the Nomenclature at the end of the paper for ease of further reference.

APPROXIMATING A CONTINUOUS RADIATION EXPOSURE BY A SERIES OF POINT EXPOSURES

To estimate the benefit of the safety system that will remove the radiation exposure, we start by calculating the effects of not removing the exposure. The change in life expectancy, $\delta X(a)$, for an individual aged, a, at the start of the notional, prolonged radiation exposure is given by equation (25) of Thomas *et al.* (2006c), repeated below

$$\delta X(a) = -cd_a e^{W(a)} \int_a^\infty e^{-W(t)} \psi_1(t-a)dt$$

(25—Thomas *et al.*, 2006c)

Where c is the risk coefficient for radiation dose rate ($Sv^{-1}y^{-1}$), d_a is the dose rate (Sv y^{-1}) and ψ_1 is the integral of the hazard function, ψ_0, as explained in equation (2) and (3) below, and where $W(t) = \int_0^t h(t)\,dt$ is the integrated hazard rate from age 0 to age t. Noting that the survival probability at age, t, is given by $S(t) = e^{-W(t)}$, we may rewrite this equation as

$$\delta X(a) = -\frac{cd_a}{S(a)} \int_a^\infty S(t)\psi_1(t-a)dt \tag{1}$$

Since the hazard rate, $h(t)$, is the base rate, unperturbed by the notional radiation exposure, it is possible to perform the integral for $W(t)$ using data from the life tables, and so produce a tabulation of $S(t)$ at each age, t. The function, ψ, is given by

$$\psi_1(t) = \int_0^t \psi_0(z)dz \tag{2}$$

where the hazard function, ψ_0, for a notional exposure period of duration, T_R, takes the form of a ramp up to a plateau (the plateau degenerates into a point in the case when $T_R = 30$ years; see figure 5 of Thomas *et al.*, 2006c) followed by a ramp back down:

$$\begin{aligned}\psi_0(z) &= (z-10)J_p(z-10) - (z-40)J_p(z-40)\\ &\quad - [z-(T_R+10)]J_p[z-(T_R+10)]\\ &\quad + [z-(T_R+40)]J_p[z-(T_R+40)]\end{aligned} \tag{3}$$

The step or 'jump' function, J_p, in equation (3) is defined in terms of the general age or time variable, x, by

$$\begin{aligned}J_p(x) &= 0 \quad \text{for } x < 0\\ &= 1 \quad \text{for } x \geq 0\end{aligned} \tag{4}$$

The hazard function, ψ_0, for a prolonged exposure may be approximated by the sum of the hazard functions, $\phi_0(z-i)$, $i = 0, 1, 2, \ldots (T_R - 1)$, for a series of point exposures of a year's worth of dose assumed to occur at the start of each year:

$$\psi_0(z) \approx \phi_0(z) + \phi_0(z-1) + \cdots + \phi_0[z-(T_R-1)] \tag{5}$$

where $\phi_0(z)$ is the function developed by Marshall *et al.* (1982) for describing a one-off nuclear exposure:

$$\phi_0(z) = J_p(z-10) - J_p(z-40) \tag{6}$$

Figure 1 shows the comparison between $\psi_0(z)$ and $\sum_{i=0}^{T_R-1} \phi_0(z-i)$ when the length of the notional exposure period is given by $T_R = 10$ years. Equation (6) may be integrated (Marshall *et al.*, 1982; Thomas *et al.*, 2007, Appendix) to give

$$\begin{aligned}\phi_1(t) &= \int_0^t \phi_0(z)dz\\ &= (t-10)J_p(t-10) - (t-40)J_p(t-40)\end{aligned} \tag{7}$$

Using equations (2) and (6), we may now integrate both sides of equation (5) to give

$$\psi_1(t) \approx \phi_1(t) + \phi_1(t-1) + \cdots + \phi_1[t-(T_R-1)] \tag{8}$$

See Figure 2 for a comparison between $\psi_1(t)$ and $\sum_{i=0}^{T_R-1} \phi_1(t-i)$, again for a notional exposure period of $T_R = 10$ years.

The matches in the two figures are generally close, although there is a tendency for $\sum_{i=0}^{T_R-1} \phi_1(t-i)$ to exceed $\psi_1(t)$ by a few percent as a result of the phase advance of the term, $\sum_{i=0}^{T_R-1} \phi_0(z-i)$, as shown in Figure 1. By equation (1), this will lead to a small overestimate of the change in life expectancy at each age and hence of the average change in life expectancy.

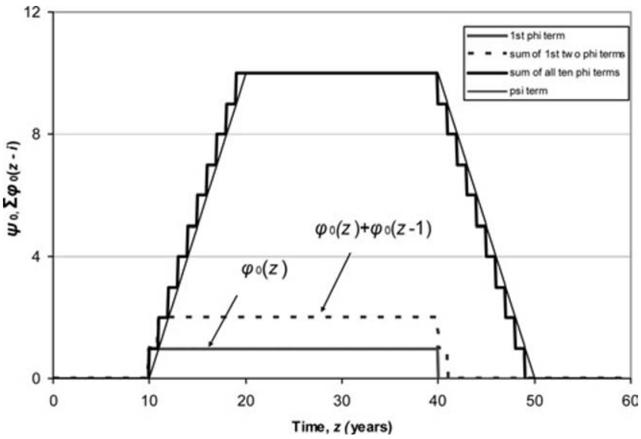

Figure 1. Comparison between $\psi_0(z)$ and $\sum_{i=0}^{T_R-1}\phi_0(z-i)$ for a notional exposure period of $T_R = 10$ years.

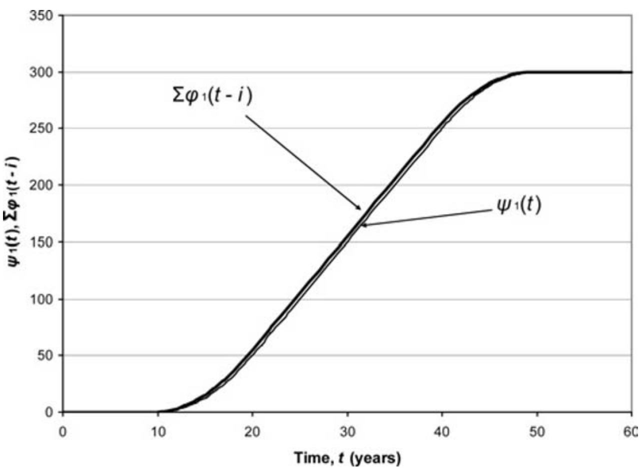

Figure 2. Comparison between $\psi_1(t)$ and $\sum_{i=0}^{T_R-1}\phi_1(t-i)$ for a notional exposure period of $T_R = 10$ years.