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Vulnerability analysis of the US power grid based on local load-redistribution

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ABSTRACT

In this paper, local load-redistribution is employed to study the vulnerability of the US power grid subjected to attacks, and the initial load of power node is assumed as the linear summation of its degree and the sum of its neighbors' degrees. Based on the simulation results, we find that with the decrease of the initial load of power node, the critical threshold of tolerance parameter increases subjected to the highest load node-based attack, but the critical threshold decreases subjected to the lowest load node-based attacks. Meanwhile, we discover that the critical threshold subjected to the lowest load node-based attack is larger than that subjected to the highest load node-based attack when the tunable parameter $\alpha \in [0.1, 0.9]$. Moreover, this paper compares the new model with the other two existing models, and the results illustrate that there are many similarities between these two existing models, but they are different from the new model presented in this paper. Furthermore, the infimum of tolerance parameter is proposed to portray the minimum cost of power grid, which can give a method to construct more robust power grids in the future.

1. Introduction

Vulnerability of power grids has been paid much attention for several years, however large scale blackouts of power grids still frequently occur in the world. For examples, the North America blackout in August of 2003 was caused by an untrimmed tree which was too close to high voltage transmission lines in the Midwest, and the failure caused redistributing power flow and resulted in large scale cascading failures; and several southern provinces of China experienced a large scale blackout in 2008, which was caused by the blizzard and resulted in enormous damages. Power grids have become the lifelines of the modern society, and its failure can cause huge damages, therefore more and more researchers have taken part in this field to investigate the vulnerability of power grids. There are many models to investigate power grids and many valuable results have been obtained. Chen et al. (2005) proposed a hidden failure model to investigate the cascading failures and mitigation assessment of power grids subjected to disturbances, Holmgren (2006) used graph model to analyze the vulnerability of electric power networks, and Chen et al. (2010) presented a hybrid method to study the attack vulnerability of power grids in terms of complex network theory. Meanwhile, Bao et al. (2009) gave a power flow entropy model to analyze the cascading failures of electric grid and Zhang et al. (2012) employed US power grid and IEEE-118 network to investigate the vulnerability of self-organizing networks.

Since complex network theory is raised in the past decades, networked models are already used to investigate the characteristics of power grids. Motter and Lai (2002) constructed a networked model to investigate the vulnerability of US power grid based on complex network theory, and the betweenness of node was assumed as the initial load of node in power grid, and the load capacity of node was assumed as a linear relationship with the initial load of node, and the authors proposed a parameter called largest connected component which was utilized to study the vulnerability of power grid subjected to cascading failures. Crucitti et al. (2004a,b) developed the model proposed by Motter and Lai (2002) to investigate the cascading failures of complex networks. Local weighted flow redistribution was adopted to investigate the robustness of the weighted networks against cascading failures (Wang and Chen, 2008), and the weight of edge ij was defined as $(k_i k_j)^{\alpha}$, where k_i and k_j being the degrees of node i and node j respectively. Li et al. (2008) presented a limited resource model of fault-tolerant capability for complex networks against cascading

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failures, and the nodes with higher loads and larger degrees were given extra capability to enhance the reliability of these nodes.

Furthermore, the vulnerability of scale-free networks was investigated according to local weighted flow redistribution (Wang et al., 2008), meanwhile the weight and load capacity of node j were defined as k_j^θ and Ck_j^θ respectively. Moreover, the local load-redistribution rule was involved into US power grid and the vulnerability was investigated by Wang and Rong (2009a,b), and the number of the fault nodes after cascading failures was applied to assess the performance changes of power grid. An interesting result that when the tunable parameter $\alpha < 0.7$, the attack to the node with the lowest load was more harmful than the attack to the node with the highest load was obtained, and this result has attracted much attention (Hines et al., 2010; Markoff and Barboza, 2010). The local load-redistribution was also applied into edge-based attacks according to a proposed breakdown probability of overloaded nodes (Wang and Rong, 2009a,b), and the properties of scale-free networks were studied. Meanwhile, the vulnerability of power grids in many countries has been also studied by many researchers (Albert et al., 2004; Crucitti et al., 2004a,b; Kinney et al., 2005). In the above networked models, there was a linear relationship between load capacity and initial load of the node, however, Dou et al. (2010) gave a model that the load capacity possessed a nonlinear relationship with the initial load of node, and they investigated the robustness of complex networks.

Based on aforementioned models, this paper presents a new methodology to construct a networked model which is applied to investigate the vulnerability of the US power grid. This paper is organized as follows. Section 2 gives a new model and introduces the other two existing models. The vulnerability of the US power grid is discussed according to different networked models in Section 3. Section 4 gives several comparative results of different models. Finally, conclusions are given in Section 5.

2. Networked model

In this section, the new networked model is presented and two existing models are also introduced to discuss the vulnerability of the US power grid. Two attack rules called the highest load node-based attacks (***HL***) and the lowest load node-based attacks (***LL***) (Wang and Rong, 2009a,b) are involved into this paper. When a node is removed from the power grid, the loads and degrees of the other nodes will be recalculated, and the nodes will be removed from the power gird one by one. Moreover, when a node collapses, its load will be redistributed to its neighbor nodes according to the proposed probability, and this is called local load-redistribution rule. In our model $\boldsymbol{R_1}$, the initial load of a node is assumed as the linear summation of its degree and the sum of its neighbors' degrees, meanwhile the load capacity of node has a linear relationship with its initial load of the node. The normalized avalanche size of nodes given by Wang and Rong (2009a,b) is employed to assess the damages of power grid subjected to attacks.

For any network, we abstract it by an undirected graph, so for any graph $G = \langle V, E\rangle,\ V = \{1, 2, 3, \ldots, N-1, N\}$ is the set of nodes, and $E = \{ij\}$ is the set of edges, the degree of a node is defined as the number of links directly connecting with the other nodes of the graph, and $k_i (i = 1, 2, \ldots, N-1, N)$ represents the degree of node i. The new networked model is presented by model $\boldsymbol{R_1}$ as follows.

$\boldsymbol{R_1}$: The initial load of node i is assumed as the linear summation of its degree and the sum of its neighbors' degrees

$$L_i = \alpha k_i + (1-\alpha)\sum_{l\in\Gamma_i} k_l \tag{1}$$

When the node i is removed from the power grid, its load will be redistributed to its neighbor nodes, and the preferential probability on redistributing the load of node i is proportional to the initial load of its neighbor node j, as precisely,

$$P_1(i\to j) = \frac{\alpha k_j + (1-\alpha)\sum_{l\in\Gamma_j} k_l}{\sum_{m\in\Gamma_i}\left[\alpha k_m + (1-\alpha)\sum_{f\in\Gamma_m} k_f\right]} = \frac{L_j}{\sum_{m\in\Gamma_i} L_m} \tag{2}$$

According to the formulas (1) and (2), the load incremental of node j after the removal of the node i can be calculated by the following formula (3),

$$\Delta L_{ij} = \frac{L_j}{\sum_{m\in\Gamma_i} L_m} \times L_i \tag{3}$$

And the load capacity of node $l (l = 1, 2, \ldots, N-1, N)$ is given as follows,

$$C_l = (1+\beta)L_l \tag{4}$$

where k_l is the degree of the node l, and Γ_i is the set of the current neighbor nodes of node i, and $\alpha \in [0, 1]$ is the tunable parameter which controls the initial loads of nodes, $\beta > 0$ is the tolerance parameter which can reflect the cost of the node.

$\boldsymbol{R_2}$: Wang et al. (2008) gave a model to investigate the vulnerability of scale-free networks. The initial load of node i was defined as follows,

$$L_i = k_i^\alpha \tag{5}$$

and the probability of local load-redistribution is given by the following formula (6),

$$P_2(i\to j) = \frac{k_j^\alpha}{\sum_{l\in\Gamma_i} k_l^\alpha} \tag{6}$$

Meanwhile the load capacity of node is the same as formula (4), where $0 \leqslant \alpha \leqslant 1$ is a tunable parameter and controls the initial load of node. Based on the model $\boldsymbol{R_1}$ and model $\boldsymbol{R_2}$, there are two critical differences, the different initial load of node L_i and the different position of the tunable parameter α.

$\boldsymbol{R_3}$: Wang and Rong (2009a,b) introduced a model to study the vulnerability of the US power grid based on two kinds of attack rules, and the initial load of node i is defined as formula (7),

$$L_i = \left[k_i \sum_{l\in\Gamma_i} k_l\right]^\alpha \tag{7}$$

and the probability of local load-redistribution is given by the formula (8),

$$P_3(i\to j) = \frac{\left[k_j \sum_{l\in\Gamma_j} k_l\right]^\alpha}{\sum_{m\in\Gamma_i}\left[k_m \sum_{n\in\Gamma_m} k_n\right]^\alpha} \tag{8}$$

The load capacity of node is also the same as the formula (4), where $0 \leqslant \alpha \leqslant 1$ is a tunable parameter.

Based on the model $\boldsymbol{R_1}$ and model $\boldsymbol{R_3}$, there are also two main differences between them, such as the different initial load of node L_i and the different position of the tunable parameter α. Moreover, the initial load of node in model $\boldsymbol{R_2}$ is also different from that in model $\boldsymbol{R_3}$. Because the initial loads of nodes in these three models are different from each other, hence they have different probabilities of local load-redistribution. Supposing the initial failure being that the node i is removed from the power grid, the load incremental of its neighbor node j can be obtained by formula (3), and the load of node j becomes $L_j + \Delta L_{ij}$. If $L_j + \Delta L_{ij} \leqslant C_j$, the power grid can recover from the disturbances, however the node j will also collapse when $L_j + \Delta L_{ij} > C_j$, and the second failure will lead to further redistributing the load $L_j + \Delta L_{ij}$, thereby the cascading failures take place.

In this paper, the vulnerability of the US power grid is investigated based on these three models, and we know that there are