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Understanding (sessile/constrained) bubble and drop oscillations

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ABSTRACT

The diffuse literature on drop oscillation is reviewed, with an emphasis on capillary wave oscillations of constrained drops. Based on the review, a unifying conceptual framework is presented for drop and bubble oscillations, which considers free and constrained drops/bubbles, oscillation of the surface or the bulk (i.e. center of mass) of the drop/bubble, as well as different types of restoring forces (surface tension, gravity, electromagnetic, etc). Experimental results (both from literature and from a new set of experiments studying sessile drops in cross flowing air) are used to test mathematical models from literature, using a novel whole profile analysis technique for the new experiments. The cause of oscillation (cross flowing air, vibrated surface, etc.) is seen not to affect oscillation frequency. In terms of models, simplified models are seen to poorly predict oscillation frequencies. The most advanced literature models are found to be relatively accurate at predicting frequency. However it is seen that no existing models are reliably accurate across a wide range of contact angles, indicating the need for advanced models/empirical relations especially for drops undergoing the lowest frequency mode of oscillation (the order 1 degree 1 non-axisymmetric 'bending' mode that corresponds to a lateral 'rocking' motion of the drop).

Contents

1. Introduction

Consider a drop or bubble; as common as a drop of rain or a pot of boiling water, with applications as diverse as aerosolized medicines or floatation of metal ores. Under the influence of surface tension, drops and bubbles adopt spherical shapes if free from constraint and external forces. In contact with a surface the constrained drop or bubble forms a shape similar to a spherical cap, but potentially deformed by external forces.

Now consider the oscillations of a drop or bubble. Deformed and/or displaced, the drop/bubble is driven back towards equilibrium by a restoring force such as surface tension, gravity, etc. As will be discussed shortly, the analytic works on drop/bubble oscillation have mainly focused on wave shapes that are axisymmetric about either an arbitrary axis for a free drop/bubble, or about the normal of the solid surface for

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a constrained drop/bubble. Recently, the pace of applied research in drop oscillations has increased, especially for the case of sessile drops. Perhaps due to the more rapid pace, some confusion seems apparent as to the precise physics governing drop and bubble oscillations, what models exist to describe them, and how such models should be applied (especially for non-axisymmetric oscillations). We believe the source of this confusion is twofold: first, due to misunderstanding of bulk (i.e. center of mass) oscillation and surface oscillation of drops and their connection to each other; and second, due to a lack of appreciation for the complexities of non-axisymmetric wave shapes and the three-dimensional character of the drop surface.

The literature on drop/bubble (especially sessile drop) oscillation is also diffuse, with no publication giving a categorizing framework for its understanding. Considering the diffuse nature of the literature available, and the different possible cases of drop/bubble oscillation, this paper first presents an overview of the present understanding of drop and bubble oscillations, including mathematical models proposed to predict resonant frequencies. This overview is categorized in terms of free versus constrained drops, and in terms of axisymmetric versus non-axisymmetric oscillations, with an emphasis on sessile (surface constrained) drops. A framework for summarizing, categorizing and understanding drop/bubble oscillations is then given. A full profile processing tool for the study of free or constrained drop/bubble oscillation is introduced and its use illustrated in analyzing new experimental results for sessile drops in cross flowing air. These new results are finally combined with those from literature to test various models given in literature.

2. Status of understanding in literature

The new experimental work presented in this paper focuses on the surface and bulk oscillations of sessile (i.e. constrained) drops driven by cross flowing air and restored by surface tension, since it is an area of interest to the authors [1]. As such, this literature review will focus on the natural surface oscillations of drops opposed by surface tension. Little literature was found on oscillations opposed by gravity, though what was found will be reviewed. Only one work was found studying (nearly) pure bulk oscillations of constrained drops [2], and no works were found studying pure bulk oscillations of constrained bubbles or free drops/bubbles, likely due to their uncommon nature as will be discussed below. This review does not consider forced oscillations in any great detail, nor the effects of surface charge/electric fields on oscillation.

2.1. Axisymmetric and non-axisymmetric surface oscillations of free drops

The oscillation of free drops (those falling/floating freely in their surroundings) has been studied since at least 1843, when Plateau reported observing them following the breakup of falling fluid streams [3]. Lord Rayleigh considered drops in vacuum, balancing surface tension and inertial forces to arrive at a formula for the frequency of axi-symmetric capillary wave oscillations of spherical drops [4]. Lord Kelvin [5] and Lamb [6] both considered oscillations balancing inertia and gravity. Gravity waves are less predominant than capillary waves for small drops as evidenced by the low Bond number (~0.14 for a 1 mm drop of water in air). Lamb also expanded the capillary analysis to drops or bubbles in an infinite medium of arbitrary density [7], and to viscous drops [6], finding that for low viscosities there is no effect on frequency. Finally, Landau and Lifshitz [8] and (briefly) Lamb [6] have considered non-axisymmetric waves. In terms of frequency, mathematical models can be categorized by whether the restoring force for oscillations is surface tension or gravity based. For surface tension restored oscillations, drop/bubble oscillation models take the form of:

$$f_{ml} = \frac{1}{2\pi}\sqrt{\frac{\gamma}{\rho_1 V \lambda_{ml,\gamma}}} \tag{1}$$

where f_{ml} is the ordinary frequency (in Hz) of an oscillation mode of degree l and order m, γ is surface tension, ρ_1 is the density of the drop/bubble, V is the volume of the drop, and $\lambda_{ml,\gamma}$ is the eigenvalue of a surface tension based drop oscillation mode of degree l and order m (a mode m-l oscillation). We give Eq. (1) in terms of volume here, since the radius of the drop is poorly defined for a non-spherical (cap) drop shape, and to allow comparison of frequencies for constrained drops of the same volume with different contact angles.

The concept of oscillation eigenvalues will be used throughout this paper since most models for drop/bubble oscillation follow the same general form. The eigenvalue technique acknowledges the similar physics governing drop/bubble oscillations in multiple cases, and facilitates comparison between the models. The eigenvalue depends on degree and order number, as stated, and can also depend on fluid properties, and contact angle or constraint type for constrained drops and bubbles. The eigenvalue does not depend on surface tension, γ. The subscripted γ in Eq. (1) instead denotes that the deformations are restored by a surface tension derived force.

For free drops/bubbles undergoing small amplitude oscillations about a spherical shape eigenvalues are given by [7,9]:

$$\lambda_{ml,\gamma} = \frac{3((l+1)+(\rho_2/\rho_1)l)}{4\pi l(l-1)(l+1)(l+2)} \tag{2}$$

where ρ_2 is the density of the fluid surrounding the drop/bubble. Eqs. (1) and (2) combined return the formulae originally derived for drops [4,6–8] and bubbles [7] by substituting $\rho_2 = 0$ and $\rho_1 = 0$, respectively.

Eq. (2) is derived from first principles by considering each oscillation mode as a spherical harmonic and using the properties of the related associated Legendre polynomials. Non-axisymmetric modes correspond to non-axisymmetric (i.e. tesseral or sectoral) spherical harmonics, resulting in parts of the drop oscillating non-axisymmetrically. As Eq. (2) shows, these non-axisymmetric modes are degenerate, in that they have the same frequency as the related axisymmetric (zonal spherical harmonic) oscillations (i.e. the eigenvalue does not depend on order, m). Eq. (2) also shows that mode m-0 is not allowed for drops since it would equate to volume change of an incompressible fluid; it is, conversely, allowed for bubbles. Mode m-1 is not allowed for either free drops or free bubbles since it would amount to movement of the bulk of the drop without any surface distortion, and therefore without any restoring force. It can thus be stated that surface oscillation of a free drop/bubble is generally decoupled from oscillation/motion of the bulk of the drop/bubble.

Subsequent researchers have advanced the field with consideration of higher viscosity inner and outer fluids (see, e.g., Morrison et al. [10] and Miller and Scriven [11], the references therein, and references in Bauer and Chiba [12]). Others have examined the combined effects of capillarity and electromagnetism [4,10], indicating that electromagnetic forces can affect (generally decreasing) the restoring force of oscillations. Non-spherical (but still axisymmetric) equilibrium drop shapes have also been considered [13–15]. Finally, large amplitude (non-linear) [15–18] axisymmetric and non-axisymmetric drop oscillations have been researched. It is suggested, at least at higher oscillation magnitudes, that the axisymmetric mode shapes are unstable, tending toward non-axisymmetric modes [18]. Natarajan and Brown [17,18] have also studied resonant energy transfer between different axisymmetric and non-axisymmetric modes of oscillation, something which in general depends on the higher order (non-linear) terms describing drop oscillation.

Consider now when gravity is the restoring force for oscillations. Then free oscillation models take the form of [5,6]:

$$f_{ml} = \frac{1}{2\pi}\sqrt{\frac{g}{R}\frac{1}{\lambda_{ml,g}}} \tag{3}$$