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Effective temperature for sheared suspensions: A route towards closures for migration in bidisperse suspension

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ABSTRACT

By extending the concept of an effective temperature, earlier introduced for sheared monodisperse suspensions, we propose a continuum theory for sheared *bidisperse* suspensions. We show the theory for sheared suspensions can be constructed from the theory for Brownian suspensions by replacing the temperature with the effective temperature. Furthermore, we explore the validity of closure relations based on mean field/free volume theory, by comparison with experimental data obtained for Brownian bidisperse suspensions. In a recent paper, we have shown that the new theory, combined with the discussed closure relations, is indeed a predictive theory.

Contents

1. Introduction

Our research group is involved in a long-term research program on the fractionation of food suspensions using membranes or microfluidic devices [1–12] The purpose of the fractionation processes is to separate bidisperse or polydisperse suspensions in several fractions of different particle composition. One physical phenomenon which induces separation in polydisperse suspensions is shear-induced migration [13].

Shear-induced migration enhances greatly fractionation processes. If a particle suspension is flowing in a narrow microchannel, a non-uniform particle density profile develops [14,15]. The particles tend to migrate towards the middle of the channel. Shear-induced migration at low particle Reynolds numbers, and if the height of the channel must be less than 50 times the particle radius to give a significant non-uniformity, $H<50a$. Furthermore, it takes a considerable length ($L>20\,H$) to have a fully developed (non-uniform) concentration

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profile [16–18]. This entrance length depends on the ratio H/a and on the bulk particle concentration.

If the particle suspension is bidisperse, the larger particles have a greater tendency to migrate. Hence, the suspension near the wall gets enriched with smaller particles [19,15,20]. In the fractionation processes one takes advantage of this enrichment. For, example one can integrate a membrane in the wall of the microchannel downstream (at a distance about equal to the entrance length), which allows passage of the smaller particles. If the transmembrane pressure is well tuned, the smaller particles permeate through the membrane, while the majority of the larger particles remain in the microchannel. A sketch of this fractionation process with the relevant length scale is sketched in Fig. 1.

Other fractionation processes split the flows at the outlet of the microchannel, via an annular insert for example — which extracts the part of the suspension enriched with the larger particles. Shear-induced migration also plays a significant role in conventional membrane separation processes [21]. It reduces the growth of a cake layer of particles that are deposited on the membrane. This effect is used in crossflow microfiltration, where strong shear flows with strain rates of 1000 s^{-1} control the height of the cake layer. A typical food applications using crossflow microfiltration is the clarification of beer, where yeast cells and aggregates are separated from the beer [9].

Designing the fractionation process, as sketched in Fig. 1, requires proper values for channel height, H, entrance length L, wall shear rate in the feed channel, and transmembrane pressure (TMP) etc. These proper values of course depend on the particle sizes and concentrations.

Shear-induced migration is reasonably well understood for monodisperse suspensions flowing in microchannels or capillaries [22–26]. However, this is not yet the case for bidisperse suspensions. There exists only one single paper on the theory for sheared, bidisperse suspensions [27]. However, this theory is phenomenological, and has little physical foundations.

Hence, for solving the design problem of the fractionation process we have been exploring shear-induced migration in the last decade by means of computer simulations [28,26]. With particle based simulations using Lattice Boltzmann we have focused on obtaining self-diffusion coefficients at finite Reynolds numbers, and bidisperse systems, which [29,30]. These particle based simulations are supposed to render closure relations for continuum models, describing the fractionation at the length scale of the membrane module [31,28]. Here, we follow the same strategy by Brady and Morris, who have investigated shear-induced diffusion at the particle scale using Stokesian Dynamics [32,33], but the migration phenomenon is investigated at the continuum level [22–25]. This multiscale simulation strategy is advantageous, because particle based simulations would take enormous computer resources to solve the migration phenomenon on the scale of a microchannel ($H\times L$) [31,28].

In a recent review paper [26], we have stated the notion that the theory for shear-induced migration in strongly sheared, monodisperse suspensions can be reformulated in terms of an effective temperature, which is linear in the viscous stress. The effective temperature is a concept from soft matter physics, used for describing the dynamics of strongly driven soft matter systems [34–37]. The concept has proven especially powerful for driven, granular media. Thermodynamic theories developed for Brownian, colloidal suspensions can often be ported to driven granular media, just by replacing the temperature with the effective temperature (a.k.a. the granular temperature) [38–40]. In driven granular media the effective temperature is linear with the square of the particle velocity fluctuations.

Inspired by the success of the effective temperature concept in granular media, researchers have tried to model sheared suspensions in a similar way [41,22,42]. In these early studies the researchers have attempted to link the particle stress tensor to the particle velocity fluctuations, which has proven to give unsatisfactory results [43]. Here, the problem is that particle velocity fluctuations give an effective temperature, srate — while the particle stress tensor is linear with the shear rate. In our review paper, we have stated that the continuum theory for sheared suspensions is more in line with thermodynamic treatment, if the effective temperature is *linear* in the shear rate. Such scaling has also been observed for other driven soft matter systems [44,45].

Using recent work of Lemaitre [46], we will show below that this linear scaling with the shear rate is due to the contribution of contact forces to the particle stress tensor. Particle velocity fluctuations give another contribution to the particle stress tensor, but become only relevant in the inertial regime. Under Stokes flow conditions, contact forces are predominantly due to hydrodynamic interactions. Using the lubrication force approximation for the hydrodynamic interactions, we show with micromechanical arguments that this leads to the proper scaling of the particle stress. Furthermore, we show its trace, the particle pressure, can be written as a product of the effective temperature and compressibility factor — similar to Brownian suspensions.

Now, that the theory for shear-induced migration of monodisperse suspensions has been put on firm footings — the route towards a theory for sheared, bidisperse suspensions appears straightforward. Here, we follow the route taken earlier for bidisperse, driven granular media [38–40].

However, for sheared suspensions other closure relations for model parameters governing the dynamics are required. In our review paper, we have analysed the existing closure relations for sheared, monodisperse suspensions. Using mean field and free

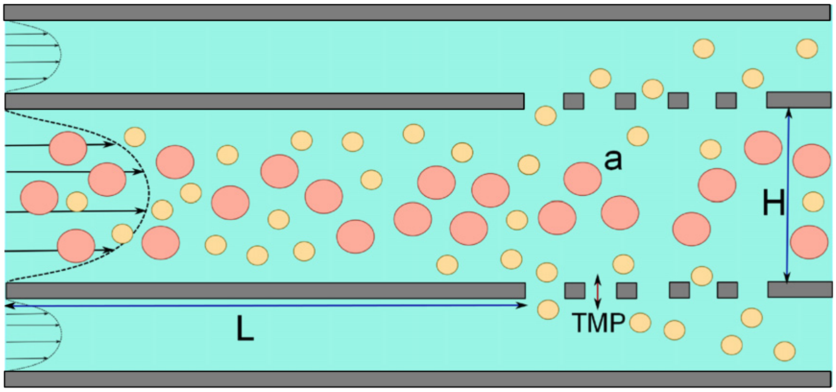

Fig. 1. Sketch of the fractionation process of bidisperse suspension using shear-induced migration in a microchannel, with height $H<50a$, and a membrane integrated in the wall downstream of the channel, at a distance larger than the entrance length, $L>20\,H$. Over the membrane there is a transmembrane pressure (TMP) which drags the smaller particles to the permeate side of the membrane.