Colloids and Surfaces A: Physicochem. Eng. Aspects 504 (2016) 407–413

Contents lists available at ScienceDirect

Colloids and Surfaces A: Physicochemical and Engineering Aspects

journal homepage: www.elsevier.com/locate/colsurfa

A novel quantitative analysis of the local deformation of the air-water surface by a floating sphere

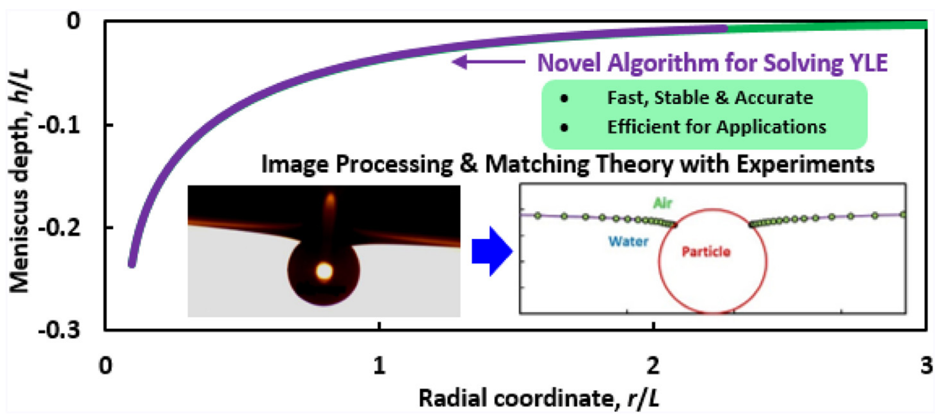
Dong-xia Feng, Anh V. Nguyen *

School of Chemical Engineering, The University of Queensland, Brisbane, Queensland 4072, Australia

HIGHLIGHTS

- Novel algorithm for solving Young-Laplace equation is proposed.
- Novel algorithm for processing and correcting interface images is developed.
- Interface deformation is quantified using the Levenberg–Marquardt algorithm.
- Interface deformation and contact angle are reliably determined.

GRAPHICAL ABSTRACT



ARTICLE INFO

Article history:
Received 26 April 2016
Received in revised form 27 May 2016
Accepted 31 May 2016
Available online 1 June 2016

Keywords:
Meniscus
Contact angle
Air-water interface
Numerical method

ABSTRACT

The Young-Laplace equation (YLE) for the deformation of external menisci governs many amazing interfacial phenomena and processes, from the walking on the surface of water by arthropods to the separation of coal and valuable minerals worth billions of dollars annually. A quantitative analysis of the phenomena suffers from problems of YLE which is a highly nonlinear differential equation with one of the two boundary conditions occurred at infinity. Available numerical solutions cannot be used for the numerical fitting of the meniscus profiles to quantify experimental results. Here we present a novel qualitative analysis involving novel numerical algorithms, which are fast and stable and, therefore, suitable for the numerical fitting to match the theoretical and experimental results to quantify the meniscus deformation. The application of the algorithms is successfully demonstrated using the experimental data for the deformation of the air-water interface around a sphere.

1. The Young-Laplace equation for quantifying the local deformation of the air-water interface by floating objects

Floating objects on the air-water surfaces are essential to many natural and industrial activities. The most natural phenomena include the climbing on the meniscus and walking of insects on the surface of lakes, ponds, rivers and the open ocean [1]. These tiny insects have characteristic dimensions of the capillary length,

* Corresponding author.
E-mail address: anh.nguyen@eng.uq.edu.au (A.V. Nguyen).

http://dx.doi.org/10.1016/j.colsurfa.2016.05.098
0927-7757/

$L = \sqrt{\sigma/\rho g} = 2.7$ mm, where $\sigma = 0.072$ N/m and $\rho = 1000$ kg/m^3 are the surface tension and density of water, and $g = 0.981$ m/s^2 is the acceleration due to gravity. The weight of their small mass (~0.01 g) is supported by the surface tension (capillary) force generated by the curvature of the free water surface which is planar on the human eyes but locally deformed around the object. Capillary forces created by the local deformation of the water surface does not only support the insects to float on the water surface but also allow them to propel themselves laterally and to climb on the air-water interface [1–3]. Industrially, understanding floating objects is critical to the flotation separation of gold, diamonds and many other valuable minerals using free water surface (called the film or surface flotation) or air bubbles (called the froth flotation) [4,5]. In these industrial applications, the particles of valuable minerals are rendered hydrophobic using a surfactant, thus capable of attaching themselves to and floating on the air-water surface while the unwanted particles remain wetted by water, do not attach to the water surface and sink to the bottom for discharge. The capillary theory of flotation originated from the work of Nutt [6] was developed [7]. It was pointed out by Nguyen and Schulze [5] that the particle weight is balanced by the buoyancy of the liquid displaced by the particle and air enclosed by the deformed meniscus (Fig. 1) as per the Archimedes Principle applied to a fully immersed object.

The local deformation of the air-water surface in supporting the flotation of small objects is governed by the capillary action which inspired many investigators for a long time, including Thomas Young of England and Pierre-Simon Laplace of France [5]. Their celebrated Young–Laplace equation (YLE) of capillary action relates the pressure difference, Δp, sustained across the interface to the surface tension and the shape of the interface as $\Delta p = 2\sigma\kappa$, where κ is the mean curvature of the air-water interface. For an external meniscus shown in Fig. 1, Δp is simply the hydrostatic pressure difference at the interface while the mean curvature can be described in terms of the second and first derivatives of the meniscus depth, h, with respect the radial coordinate, r. YLE can be described as follows:

$$\rho g h = \sigma \left\{ \frac{d^2h/dr^2}{\left[1 + \left(dh/dr\right)^2\right]^{3/2}} + \frac{dh/dr}{r\left[1 + \left(dh/dr\right)^2\right]^{1/2}} \right\} \quad (1)$$

where ρ is the difference between the water and air densities.

Solution to YLE requires two boundary conditions. They are as follows:

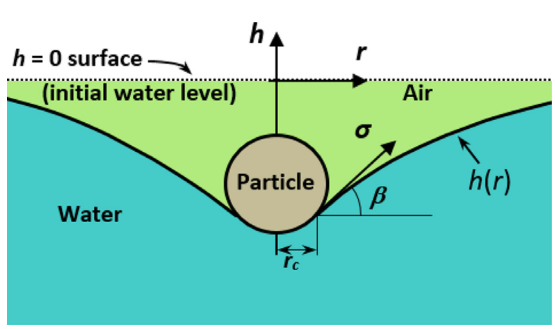


Fig. 1. An (external) meniscus formed by the deformation of a water surface (infinite plane) by a floating sphere. Due to the rotation symmetry, a reduced cylindrical coordinate system (r, h) can be used to describe the deformation. β is the angular inclination of the air-water meniscus at the three-phase contact towards the horizontal plane. σ is the air-water surface tension. r_c is the radius of the contact point.

(1) The boundary condition at the triple contact points, $r = r_c$, on the particle surface,

$$\left(\frac{dh}{dr}\right)_{r=r_c} = \tan\beta \quad (2)$$

(2) The boundary condition at infinity,

$$\left(\frac{dh}{dr}\right)_{r\to\infty} = 0. \quad (3)$$

An additional condition, which is $h = 0$ as $r \to \infty$, is already incorporated in Eq. (1).

YLE described by Eq. (1) is a highly nonlinear differential equation. Its numerical solution has been the interest of many investigators since the mathematical challenge involves a two-point boundary-value problem with one boundary value at infinity and cannot be directly solved using the traditional numerical methods, such as the Runge-Kutta methods, designed to solve initial-value problems [8,9]. Indeed, Huh and Scriven [11] transformed the two-point boundary-value problem into an initial-value problem by splitting the profile of the meniscus into two parts. The splitting point was chosen at a finite radial distance, r^*, but sufficiently far away from the triple contact line to ensure that the far-field part of the meniscus profile had a small slope ($dh/dr \ll 1$) and, thus, could be calculated using an approximate solution to Eq. (1) such as the celebrated Poisson solution—see Eq. (8). A typically chosen slope was 0.5°. Using the approximate prediction for the meniscus depth and the slope at the splitting point, the second part of the meniscus profile was obtained by the numerical integration of Eq. (1) back to the contact point using the Runge-Kutta solver. The results is a set of families of solutions, parameterized by the radial coordinate of the splitting point and the slope of 0.5°. The numerical data may be collected in the tabular form [10].

Rapacchietta and Neumann [8] proposed the shooting method to solve the two-point boundary-value problem. The shooting started with a guess for the maximum depth of the meniscus at the contact point. It then integrated Eq. (1) with the initial condition given by Eq. (2) to a faraway point but of finite radial distance, where Eq. 3 was met with a small tolerance. The improved guess was then obtained using the Newton iteration method and was repeated until a stable solution was achieved. Unfortunately, we found that the initial-value problem described by Eq. (1) and (2) were ill-conditioned, and, therefore, the shooting method was very unstable, especially when overshooting occurred, leading to a positive value for the meniscus depth which would be the physically unreal. Also, the tabulated numerical results obtained by Huh and Scriven's approach usually require an interpolation scheme for meeting the condition described Eq. (2). This interpolation is often inaccurate, cumbersome, and tedious. The available results for YLE for the meniscus deformation are not suitable for quantifying the local deformation of the air-water surface in supporting the floating spheres.

The aim of this paper is to present a simple and stable yet efficient method to solve the Young-Laplace equation. The method is suitable for quantifying the meniscus deformation and flotation of spheres at the air-water free surface, through fitting the numerical results with the experimental data.

2. Method of solution and scheme of implementation

Solving Eq. (1) by the integration with respect to r suffer a serious problem of blow-up when there is a neck of the deformed meniscus because at the neck the first derivative of the meniscus depth with respect to radial distance approach infinity, i.e., $dh/dr \to \infty$. To avoid this blow-up problem, Eq. (1) can be parametrized and integrated using the arc length of the meniscus or the angle ϕ of inclination of the meniscus. Here we conveniently use the angle of