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# Effects of slip velocity on rotating electro-osmotic flow in a slowly varying micro-channel

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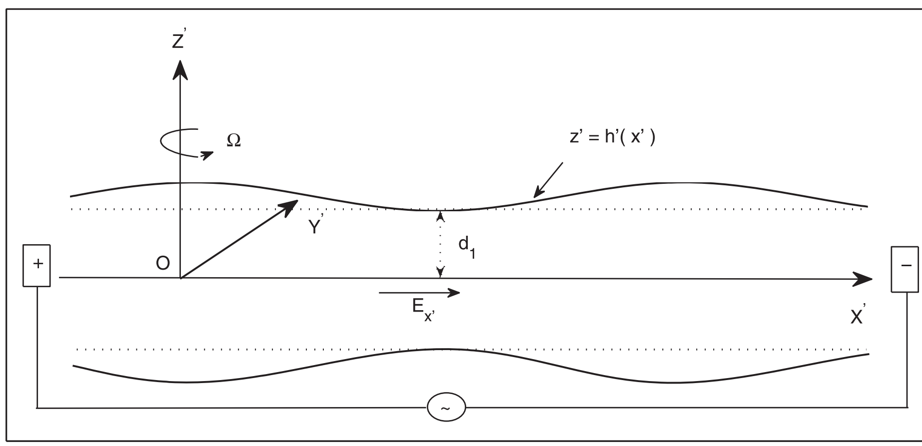
## HIGHLIGHTS

- Effect of slip velocity on rotating electroosmotic flow is studiedin a micro-channel.
- The electro-osmotic flow takes place in a wavy micro-channel.
- The problem is solved numerically by developing Crank–Nicolson scheme.
- Rotating Reynolds number plays an important role in controlling EOF.
- Higher zeta potential can facilitate the transport of fluids in a microfluidic devise.

## GRAPHICAL ABSTRACT




Physical sketch of the problem.

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## ABSTRACT

In this paper, we have studied the effects of slip velocity on rotating electro-osmotic flow in a non-uniform micro-channel. Electro-osmotically driven fluid flow takes place in a micro-channel with flexible walls in a rotating system. The potential electric field is applied along the length of the micro-channel describing the Poisson–Boltzmann equation. We assume that the entire system is rotating about the height of the channel. The non-linear Poisson–Boltzmann equation is solved numerically based upon which a Crank–Nicolson numerical scheme is developed for obtaining velocity distribution. With an aim to validate our numerical results a comparison has been made with the previous study in the case of no-slip condition and found to be good agreement.



## 1. Introduction

It is well known that electro-osmotic flow (EOF) is an alternative process of transporting fluids induced by an applied electric field. The pressure driven flow in many microfluidic devices such as micropumps and microvalves are complicated to design and

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fabrication and they are prone to mechanical failure or fabrication defects [1]. Electro-osmotic flow (EOF) takes place in a microchannel with response to an axially electric field, which results from the motion of mobile counter-ions in the diffuse portion of the electrical double layer (EDL). The electro-osmotic flow has wide range of applications in micro- and nanofluidic based lab-on-a-chip, biomedical lab-on-a-chip devices, biochemical processes, mixing of fluids, species separation, DNA sequencing/analysis as well as biological/chemical agent detection sensors and many others. Recently microfluidic devices are also applied in microbiological sensor and micro-electromechanical system (MEMS) due to rapid change of BioMEMS.

The effect of rotation on electro-osmotic flow has been the very new topic of the research in the microfluidic system to enhance the equality of mixing liquids. It may effectively reduce the time of mixing. The generation of vortices in electro-osmotic flow depends on the geometry/shape of the micro-channel. In this regard, Ajdari [2] demonstrated the generation of vortices in electro-osmosis with sinusoidal surface potential in micro-channel. Takashima [3] theoretically examined the effect of rotation in a dielectric fluid confined between two parallel plates under the action of an AC electric field by neglecting centrifugal force. Chang and Wang [4] pointed out that the electro-osmotic flow (EOF) may be situated in a rotating system such as in centrifuges for controlling flow and species separation. They investigated the rotating EOF in a micro-channel bounded by two parallel charged plates. In their study, it is observed that the location of maximum velocity shifted towards the boundary with increasing rotation. Xie and Jian [5] numerically studied on the rotating electro-osmotic flow of power-law fluids in a slit micro-channel in the presence of high zeta potential at the walls by using non-linear Poisson–Boltzann potential distribution. They reported that the direction of velocity significantly alters in the presence of large rotating angular velocity due to the effect of coriolis force on fluid flow. Li et al. [6] have investigated the rotating electro-osmotic flow of an incompressible third grad fluid in a micro-channel. They obtained approximate analytical solutions of the flow velocity and flow rate by using perturbation technique based on the linearized Poisson–Boltzmann equation. Most of these studies are restricted to flow in the channels of uniform cross-section with no-slip velocity conditions and constant zeta potential. However, upto date, few number of research works [2,7–9] conducted electro-osmotic flow in a channel with asymmetric or non-uniform charged walls. The combine pressure-driven and electro-osmotically driven flow of a power-law fluid through a slit micro-channel bounded by asymmetric walls with periodic variation in shape and zeta potential was carried out by Qi and Ng [7].

An interfacial slip due to hydrophobic interactions is generally represented by a Navier slip condition, in which the slip velocity is expressed as a function of the normal gradient of the tangential velocity component. The velocity slip at the liquid–solid interface plays an important role for electro-osmotic flow through a micro-channel in the depletion layer. The velocity of the depletion layer yields the apparent hydrodynamic slip [10]. This phenomenon is observed in the polymer solutions [11,12], colloidal particles [13] and significantly in red blood cells suspensions [14–16]. Berli and Olivares [10] and Goswami and Chakraborty [17] theoretically studied the electro-osmotic flow in a micro-channel with the interfacial slip condition. Zhao and Yang [18] have investigated the combined effects of streaming potential and hydrodynamic Navier slip on pressure-driven flows in micro-channel. They presented analytical solution by invoking Debye-Hückel linearization and derived a formula for critical slip length for designing hydrophobic surfaces. However, no study has yet been reported in the scientific literatures for analyzing the effect of hydrodynamic slip on rotating electroosmotic flow in microfluidic systems.

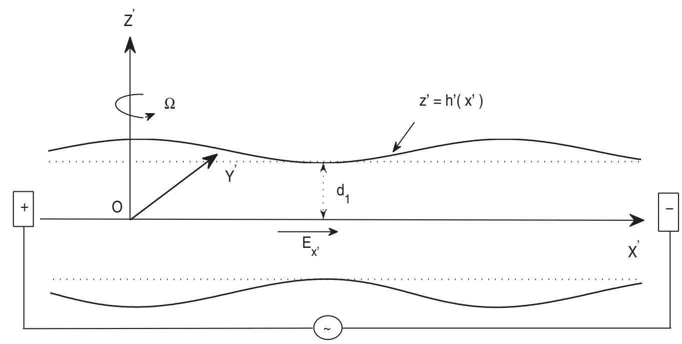


**Fig. 1.** Schematic representation of the physical problem.

In this work, we have developed a numerical model for studying the effects of slip velocity on rotating electro-osmotic flow in a micro-channel, whose walls are slowly varying periodically. The problem is formulated without imposing pressure gradient and assuming that the flow Reynolds is very small in the case of microfluidic application. The zeta potential is taken into account as it is an important parameter for electro-osmotic flow phenomenon. The potential distribution is obtained by solving the Poisson-Boltzmann equation. However, the Debye-Hückel linearization approximation is not invoked in this study in order to obtain the flow behaviour in the presence of zeta potential. Finally, the numerical solution to the axial and transverse velocity components have been obtained and the computed results are analyzed for the effects of slip parameter, rotational Reynolds number, electro-osmotic parameter as well as the amplitude of oscillation of channel walls. Moreover, a comparison figure has been presented with an aim to validate our numerical model with those obtained analytical solutions.

## 2. Mathematical formulation

### 2.1. Geometrical model

The rotating electro-osmotic flow of fully developed incompressible bio-fluid flow through a negatively charged non-uniform micro-channel is considered. The schematic representation of the physical problem is shown in Fig. 1, where $(x', y', z')$ are the cartesian co-ordinates. The flow is assumed to be symmetric about the $x'$-axis and flowing in the $x'$ direction. The height of the channel is bounded by $z' = \pm h'(x')$ which is assumed to vary sinusoidally. The geometrical expression of the wavy channel is given by

$$h'(x') = d_1 + a_1 \cos\left(\frac{2\pi x'}{L}\right), \tag{1}$$

where $d_1$ is the constant height of the channel and $a_1$ is the amplitude of the wavy walls and $L$ is the finite and large length of the micro-channel. The potential electric field is applied along the $x'$ direction which provides the necessary driving force for the electro-osmotic flow. The height of the micro-channel is considered to be much smaller than its length and breadth. We assumed that the entire system rotes about the $z'$- axis with angular velocity $\Omega$. For the sake of simplification, the overlap electrical double layers at the micro-channel and wavy effects of the walls are not considered. We also assume that the fluid properties are independent of temperature without imposing pressure gradient.

### 2.2. Electrical potential distribution

Using the basic theory of electrostatics, the net electric charge density $\rho_e$ in the diffuse layer of EDL is coupled with the potential