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Rotating electro-osmotic flow of third grade fluids between two microparallel plates

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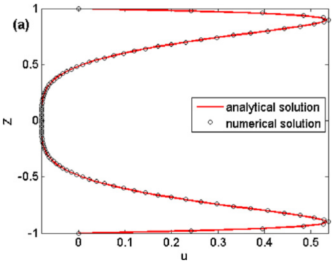
HIGHLIGHTS

- Rotating EOF of third grade fluids between two micro-parallel plates is analyzed.
- Analytical rotating EOF velocity is obtained by perturbation method.
- Finite difference method is used to give the numerical rotating EOF velocity.
- The influences of related parameters on the rotating EOF velocity are discussed.

GRAPHICAL ABSTRACT



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ABSTRACT

In this paper, a rotating electro-osmotic flow (EOF) of an incompressible third grade fluid between micro-parallel plates is considered. The analysis is based upon the modified Navier–Stokes (N–S) equation for a transport electrolyte in a rotating frame of reference. Within the framework of linearized Poisson–Boltzmann (P–B) equation, exact solutions of the electric potential distribution are given. The approximate analytical solutions of the velocity and the flow rate are achieved by using perturbation techniques under the assumption of small non-Newtonian parameter Λ. In addition, the finite difference method is used to obtain the numerical solutions. We compare the analytical solutions with the numerical solutions within admissible parameter range, and obtain a good agreement between these solutions. The influences of the Reynolds number Re, the dimensionless non-Newtonian parameter Λ, the dimensionless electro-kinetic width K on the rotating EOF velocity profiles and volume flow rate are investigated in detail. With the increasing Reynolds number Re and non-Newtonian parameter Λ, the velocity is decreasing. The large angular velocity has an obvious effect near the center core of the micro-channel and causes a reversal in the velocity direction.

1. Introduction

Microfluidics is one of the most important research areas in micro-electro-mechanical-systems (MEMS) due to its various applications, such as biochip, micro-pump and medical sensors [1,2]. Due to the effect of ionization, ion adsorption or ion dissolution, the solid surfaces may develop electrostatic charges when brought in contact with an aqueous solution. These electrostatic charges on the solid wall repel coions and attract counterions. The counterions assemble on the solid surface and then form the stern layer with a thickness of several ionic diameters. Next to the stern layer, the diffuse layer is formed, which contains both positive ions and negative ions and the ion density satisfies the Boltzmann distribution [3]. The Stern layer and the diffuse layer constitute the

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so-called electrical double layer (EDL). The free ions migrate under the influence of an external electric field along the solid wall surface and carry a bulk motion of the liquid by viscosity. This type of flow is called as the electro-osmotic flow (EOF). The EOF has wide applications in chemical, biological analyses and medical diagnoses such as dewatering waste sludge, membrane separation and removing poisonous heavy metal ions from soils. In the field of microfluidics, the EOF attracts more attentions due to many operational advantages, such as better flow control, high reliability and low noise, etc.

Extensive studies of EOF of Newtonian fluids have been conducted in micro-channels [4–7]. However, microfluidic devices are often used to analyze non-Newtonian biofluids in most of the physiology and medicine applications. The relationship between dynamic viscosity and shear rate should be constructed for the constitutive equation of non-Newtonian fluids. Various studies of EOF of non-Newtonian fluids have been conducted to explain the complex flow behavior in micro-channels. Zhao et al. [8] presented the EOF of power-law fluids between two parallel plates. The variations of the shear stress, dynamic viscosity and velocity distribution with fluid behavior index were discussed in detail. Analytical solutions for velocity, temperature and concentration distributions were given by Das and Chakraborty [9] for the EOF of power-law liquids. Jian et al. [10] studied the AC EOF of Maxwell fluids in a rectangular micro-channel and obtained analytical solutions of the velocity profiles and volumetric flow rates. The AC EOF of generalized Maxwell fluids between two micro-parallel plates was investigated by Liu et al. [11].

As one kind of the non-Newtonian fluids, Rivlin–Ericksen fluids received much attention in the literature [12]. The second and third grade fluids belong to a subclass of the differential type Rivlin–Ericksen fluids. Dunn and Rajagopal [13] gave a complete discussion on the second and third grade fluid and established several new results concerning the thermodynamics of these materials. Benharbit and Siddiqui [14] studied the planar motion of the second grade fluid for steady and unsteady cases. The flow of a generalized second grade fluid between heated plates has been reported by Gupta and Massoudi [15]. Although the second grade model is able to predict the normal stress differences, it does not respond to shear thinning or thickening properly due to its constant apparent shear viscosity. The grade three or four model exhibits shear-dependent viscosity. For this reason, some studies may be well described by the fluids of grade three or four. In practice, blood [16] and STP (Scientifically Treated Petroleum) motor oil additive consisting mainly of a polyisobutylene polymer dissolved in petroleum [17] can be described by third grade fluid model. Daivis et al. [18] computed the coefficients of the third grade fluid by employing non-equilibrium molecular dynamics simulations. Akgül and Pakdemirli [19] presented the EOF of the third grade fluids between micro-parallel plates. The influences of non-Newtonian parameter, Joule heating effect and viscosity index on the velocity and temperature profiles were analyzed in detail.

In fact, the microfluidic system may be located in a rotating environment, such as in centrifuges for mass separation and flow control. Rotating flow is a classical subject and there are many experimental, theoretical and numerical works about the rotating effects on the transport of the Newtonian and non-Newtonian fluids. The EOF system may be situated in a rotating frame. A good advantage of using a centrifuge in the microfluidic network is that the centrifugal pump also works to dispel bubbles of gas. Duffy et al. [20] conducted experimental investigation of the microfluidic system in which fluids are pumped by centrifugal force. The rotating EOF of the Newtonian fluids has been considered by Chang and Wang [21], they obtained the analytical expressions of velocity profiles and discussed the effects of the angular velocity on the velocity and the flow rate. Takashima [22] discussed the effect of rotation on electrohydrodynamic instability under the simultaneous action of a vertical ac electric field and a vertical temperature gradient. Ruo et al. [23] reported the effect of rotation on the electrohydrodynamic instability of a fluid layer with an electrical conductivity gradient. Othman [24] considered the electrohydrodynamic instability in a rotating viscoelastic dielectric fluid layer with heating effect from below. They found that the uniform rotation has a stabilizing effect on the onset of the instability for the cases with stress-free boundary conditions. Recently, Xie and Jian [25] numerically studied the rotating EOF of power-law fluids between two micro-parallel plates at high zeta potentials by using finite difference methods. The influences of flow behavior index n and the angular velocity on velocity profiles and volume flow rates were explained.

The purpose of this article is to further expand the studies of Chang and Wang [21] and Xie and Jian [25] and considered the rotating EOF of third grade fluids through two micro-parallel plates. The Navier–Stokes equation is modified based on the constitutive relation of third grade fluids in the rotating frame. The approximate analytical solutions are obtained by perturbation method by assuming small non-Newtonian behavior. Furthermore, the numerical solutions are compared with the approximate analytical solutions and a good agreement with each other can be reached.

2. Model description

In this section, we analyze the rotating EOF of the third grade fluids between two infinite micro-parallel plates. The selection of the coordinate system (x^*, y^*, z^*) is illustrated in Fig. 1. An electrical field is applied along the x^* axis of the channel. The y^* axis is pointing toward the inside of the plane and the z^* axis is vertical to two plates separating by a distance of $2h$. The whole system is rotating with a constant angular velocity Ω about the z^* axis.

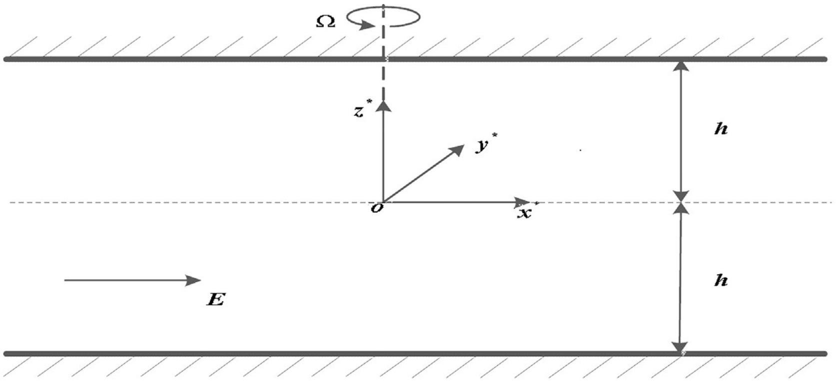

Fig. 1. Schematic of rotating EOF of third grade fluids between two infinite microparallel plates.