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# Foam improved oil recovery: Foam front displacement in the presence of slumping

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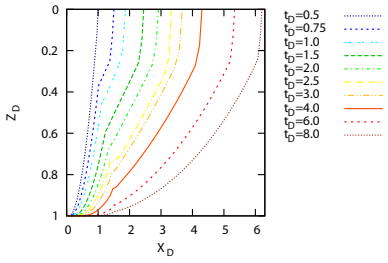
## HIGHLIGHTS

- Shape of foam front during foam improved oil recovery is modelled.
- Foam more mobile at top as dense surfactant solution slumps downwards.
- Foam front develops transient concavity but this migrates toward bottom of the front.
- At long times, foam front actually develops convex kink.
- Apparent horizontal propagation velocity is uniform across convex kink.

## GRAPHICAL ABSTRACT

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## ABSTRACT

Foam is often used in improved oil recovery processes to displace oil from an underground reservoir. During the process, the reservoir is flooded with surfactant, and then gas is injected to produce foam in situ, with the foam front advancing through the reservoir. Here the effect of surfactant slumping (downward movement of surfactant in relation to a lighter phase) upon the advance of a foam front is presented. Slumping which can be associated with foam drainage, coarsening and collapse, causes a rise in mobility of the foam front specifically near the top of the front. The description of a foam front displacement for an initially homogeneous foam mobility is therefore modified to account for slumping-induced inhomogeneities. Numerical solution for the front shape shows that, although slumping transiently produces a localised concave region on the otherwise convex front, this concavity has little effect on the long term front evolution. In fact in the long-time limit, a *convex* kink develops on the front: an analytical solution describing the convex kink agrees very well with the numerics.



## 1. Introduction

Surfactant and gas are often injected into underground oil reservoirs so as to produce foam that can subsequently displace oil and achieve improved oil recovery: under these circumstances engineers wish to predict how the foam front advances through the reservoir displacing the oil. In the calculations of foam front movement through an oil reservoir an idealised model for so called surfactant-alternating-gas (SAG) injection [1,2], is used. The model is known as 'pressure-driven growth' [3].

In what follows we introduce the 'pressure-driven growth' model with a minimum of detail as it is already well discussed in literature [1–3]. However the appendix provides additional detail about the model for readers who require it.

During the surfactant-alternating-gas (SAG) process a foam bank advances into an oil reservoir that has already been flooded

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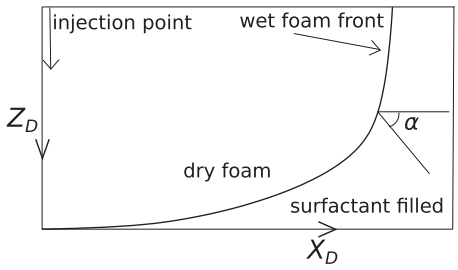


**Fig. 1.** Schematic of a foam front displacement. The region to the right of the front is filled with liquid (surfactant solution): this is the so called 'liquid bank'. The region to the left of the front up to the gas injection point is filled with dry foam with comparatively large bubbles: this is the so called 'gas bank' or 'foam bank'. The wet foam region at the front itself (where bubbles are comparatively small) provides the dominant resistance to motion. The horizontal coordinate $X_D$ and vertical coordinate $Z_D$ (measured downwards) and the angle $\alpha$ are indicated on the figure.

with surfactant in aqueous solution (the so called liquid bank). The pressure driving the foam is the difference between the injection pressure and the hydrostatic pressure (the latter of course varying with depth). The pressure-driven growth model assumes (with some justification from so called fractional flow theory [1]) that most of the resistance to the foam motion arises from a region of wet foam located right at the foam front, and moreover this resistance grows according to the distance that the front has displaced (because the wet foam region gradually thickens over time). Balancing the driving pressure force with the resistance leads to a prediction for the speed of material points on the foam front, the direction of motion being normal to the front.

Mathematically the model consists of a system of differential equations for the motion of the material points, the equations being solved numerically [2], in our case with an algorithm programmed in Matlab.

The system of equations can be conveniently written in dimensionless form, using scales identified by [1] (see also Appendix A). The dimensionless equations describing the system (1)–(3) are shown as follows:

$$\frac{dX_D}{dt_D} = \frac{(1 - Z_D)\cos\alpha}{s_D} \tag{1}$$

$$\frac{dZ_D}{dt_D} = \frac{(1 - Z_D)\sin\alpha}{s_D} \tag{2}$$

$$\frac{ds_D}{dt_D} = \sqrt{\left(\frac{dX_D}{dt_D}\right)^2 + \left(\frac{dZ_D}{dt_D}\right)^2} \tag{3}$$

where $X_D$ gives horizontal position in a rectangular reservoir, $Z_D$ the vertical position (measured downwards from the top), $t_D$ the time, $s_D$ the distance the front travels, and $\alpha$ the angle giving the front orientation as depicted in Fig. 1 (specifically $\tan\alpha = -dZ_D/dX_D$, implying that Eqs. (1)–(3) are partial differential equations in space and time, rather than ordinary differential equations in time only). Note that $Z_D = 1$ corresponds to the point at which hydrostatic pressure balances injection pressure: the front cannot advance to depths beyond $Z_D = 1$.

The initial and boundary conditions are:

$$X_D(Z_D, 0) = 0 \tag{4}$$

$$s_D(Z_D, 0) = 0 \tag{5}$$

$$\alpha(0, t_D) = 0. \tag{6}$$

In the implementation of the initial conditions in the computer program, those given by Eqs. (4) and (5) are changed to $X_D(Z_D, 0) = s_D(Z_D, 0) = s_{D_0}$ where $s_{D_0}$ is a small parameter (typically chosen here to be $s_{D_0} = 0.001$) to avoid having infinite values of $dX_D/dt_D$ and $dZ_D/dt_D$ at $t_D = 0$. Starting from a vertical front, for a homogeneous medium (i.e. a homogeneous reservoir, with in addition, foam mobility being homogeneous along the front), the system tends to give a convex shape for the front at finite time: the top of the front advances more than parts lower down because the difference between injection pressure and hydrostatic pressure is greater at the top.

Boundary condition (6), which according to the definition of angle $\alpha$ says that material instantaneously at the top of the reservoir is moving parallel to the top, implies also that the top of the front has unit speed at the particular time when it has displaced by unit distance. The volume swept by the front grows over time, albeit this rate of sweeping volume slows down as time proceeds.

Numerical solution for pressure-driven growth, i.e. discretising the front shape and computing the evolution of the discretised front material points, is known to present challenges [2,3]. In our numerical implementation it is possible to trace the front displacement until large times, and still have a fair representation of its shape, because a rule has been implemented for subdividing front segments whenever they become too long, which occurs particularly near the top of the reservoir. Such a rule is given by Eq. (7), where the subscript 0 indicates a point at the top of the reservoir, the subscript 1 refers to the next point below the top, and $n$ indicates a new point between these two.

$$X_{D_n} = X_{D_0} - \left(\frac{Z_{D_0} - Z_{D_n}}{Z_{D_0} - Z_{D_1}}\right)^{3/2} \left(X_{D_0} - X_{D_1}\right). \tag{7}$$

This equation respects a known mild singularity [3] for the front curvature at the top boundary: specifically it has been shown [3] that in the limit of very small $Z_D$ values, the amount that $X_D$ falls behind the leading edge at the top of the reservoir is proportional to $Z_D^{3/2}$, corresponding to a curvature scaling proportional to $Z_D^{-1/2}$. Further details about this singularity and why it arises are given in Appendix A.

As already mentioned above, the foam front (at least for a homogeneous system) is expected to have a convex shape. This is somewhat fortunate, because concavities are known to be extremely problematic when implementing pressure-driven growth [3]. Nevertheless, even small concavities in the shape of the curve caused by numerical artifacts (for example, truncation and/or round-off error accumulation) or by the physical nature of the system (e.g. if the medium is not perfectly homogeneous) could lead to a completely distorted shape of the curve, as concave points tend to fall increasingly far behind their neighbours [3].

The evolution of concavities over time, if they are not addressed, causes the formation of spurious loops [3] (regions where points of the front cross over each other). Spurious loops are generated whenever the points towards the back of a concavity are left behind but the points further ahead converge towards each other and cross over one another. Points towards the back of a concavity can have their velocities corrected to avoid them falling behind (the corrected version represents the physics that we want the pressure-driven growth model to capture [3], specifically the backs of the concavities constitute 'shocks' which have a different speed from front material points which constitute 'characteristics' of the governing partial differential equations). The velocity correction does however cause the length of the intervals towards the back of the concavity to decrease over time, in turn requiring implementation of a rule to eliminate short intervals. For these reasons, it is necessary that the algorithm be able to deal with both concavities and shrinking intervals.

Our way to handle the presence of concavities in the front shape is, as mentioned above, by applying a correction in the velocity calculations (the correction being reflected in the time derivatives, Eqs. (1) and (2) in the original system) when certain criteria on the degree of concavity are met. This aims to speed up the displacement