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Non-locality and viscous drag effects on the shear localisation in soft-glassy materials

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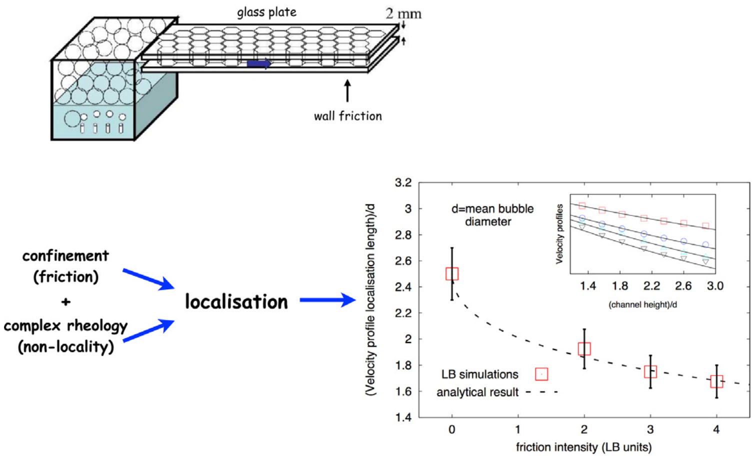
HIGHLIGHTS

- We studied analytically and numerically Couette flows of quasi-2d foams/emulsions.
- We focused on the effect of cooperativity and wall friction on the velocity profiles.
- We provided asymptotic expressions for Bingham fluids close to or well above yield.
- We tested the analytical results against numerical data showing good agreement.

GRAPHICAL ABSTRACT



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ABSTRACT

We study the Couette flow of a quasi-2d soft-glassy material in a Hele–Shaw geometry. The material is chosen to be above the jamming point, where a yield stress σ_Y emerges, below which the material deforms elastically and above which it flows like a complex fluid according to a Herschel–Bulkley (HB) rheology. Simultaneously, the effect of the confining plates is modelled as an effective linear friction law, while the walls aside the Hele–Shaw cell are sufficiently close to each other to allow visible *cooperativity* effects in the velocity profiles (Goyon et al., 2008). The effects of cooperativity are parametrized with a steady-state diffusion–relaxation equation for the fluidity field $f = \dot{\gamma}/\sigma$, defined as the ratio between shear rate $\dot{\gamma}$ and shear stress σ. For particular rheological flow-curves (Bingham fluids), the problem is tackled analytically: we explore the two regimes $\sigma \gg \sigma_Y$ and $\sigma \approx \sigma_Y$ and quantify the effect of the extra localisation induced by the wall friction. Other rheo-thinning fluids are explored with the help of numerical simulations based on lattice Boltzmann models, revealing a robustness of the analytical findings. Synergies and comparisons with other existing works in the literature (Barry et al., 2011) are also discussed.

1. Introduction

In a wide variety of systems, such as emulsions, foams, and granular materials [1–4], when the packing fraction of elementary

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constituents (droplets, bubbles, grains) exceeds a critical value, dynamical arrest occurs and the system undergoes a kind of transition, known as *jamming*. Above the jamming point, a yield stress σ_Y emerges, below which the material deforms elastically and above which it flows like a complex fluid. Upon confinement and increase of the droplets/bubbles/particles concentration, a challenging question concerns the role of microscopic plastic rearrangements and the emergence of their spatial correlations exhibiting *cooperativity* flow behavior at the macroscopic level [5–9]. Such rearrangements affect the overall rheological behaviour of the material, usually described by the Herschel–Bulkley (HB) law, relating the stress σ to the shear rate $\dot{\gamma}$. Goyon et al. [5] have demonstrated that a modification of the local continuum theory can be successful in accounting for the observed experimental velocity profiles of concentrated emulsions. In particular, they introduced the concept of a spatial cooperativity lengthscale ξ and postulated that the fluidity, defined as $f=\dot{\gamma}/\sigma$, is proportional to the rate of plastic events [9] and follows a diffusion–relaxation equation when it deviates from its bulk value

$$\xi^2\Delta f(\mathbf{r})+f_b(\sigma(\mathbf{r}))-f(\mathbf{r})=0. \tag{1}$$

The quantity f_b is the bulk fluidity, i.e. the value of the fluidity in the absence of spatial cooperativity ($\xi=0$). The non-local Eq. (1) has been justified [9] based on a kinetic model for the elastoplastic dynamics of a jammed material, which takes the form of a non-local kinetic equation for the stress distribution function. In the steady state, under the hypothesis of weak cooperativity, the model predicts non-local equations of the form (1), plus an equation predicting a proportionality between the fluidity and the rate of plastic events. This picture was later applied to other complex fluids, such as Carbopol gels [10], granular media [3,11], and foams in a 2d cylindrical Couette geometry [8]. The spatial cooperativity was shown to be of the order of a few times (typically five) the size of the elementary microstructural constituents, i.e. the droplets for emulsions [5,6,12], the bubbles for foams [8], the blobs for a polymeric gel [10]. The fluidity model agrees with existing experiments, and provides a convenient framework to rationalize the flow of confined complex fluids. However, at least two points remain unclear and largely unexplored. First, the issue of the boundary condition at solid walls for *f*. Only recently, Mansard et al. [13] explored the role of surface boundary conditions for the flow of a dense emulsion. Both slippage and wall fluidization were shown to depend non-monotonously on the roughness. Second, the fluidity parameter *f* has been seldom related to an independent and direct measure of the local density of plastic events. Sometimes, indirect indications of such a relation have been proposed, based on the correlations of the fluctuations of the shear rate [7]. Using numerical simulations based on the bubble model [14], Mansard et al. [15] were able to measure independently the fluidity and the density of plastic events, but they show that the two quantities are not proportional; more precisely, the rearrangement rate was found to be a sublinear power (with an exponent 0.4) of the fluidity. On the other hand, using experiments in a Hele–Shaw cell and simulations based on lattice Boltzmann method, we showed recently [16] that for foams and emulsions flowing in a 2d channel, there is a good correlation between the rate of plastic events and the fluidity.

Very frequently some of the systems of interest are confined so as to be quasi-2d: this is the case of Hele–Shaw cells [17–19], or quasi-2d systems made of bubbles confined between a plate and a liquid surface [20]. A friction force due to the presence of one or more confining plates may provide *shear localisation* for the velocity profiles on lengthscales which can be of the order of a few bubble/droplet sizes, thereby interfering with the cooperativity lengthscale described above. This extra localisation is usually parametrized with another lengthscale related to the viscosity and wall friction [21]. This naturally poses the question on how to rationalize the coupled role of friction and non-locality. Barry et al. [22] combined the non-local constitutive equation for the fluidity field (1) with the continuum theory of 2d shear localisation for a foam in a Couette Flow [21]. They showed that the localisation length due to friction is increased by cooperativity, and explored the limiting cases of zero and infinite cooperativity length. Due to the generality of their formulation, their analysis may be directly applicable to other complex fluids.

The aim of this paper is to complement the results by Barry et al. [22] exploring the complex flow of a soft-glassy material in a Hele–Shaw geometry with both friction and non-locality. The problem is tackled analytically for the case of a Bingham fluid, where we study the two regimes $\sigma\gg\sigma_Y$ and $\sigma\approx\sigma_Y$. A distinctive feature of our analysis, is to explore those situations where the wall acts as a source of fluidity propagating into the bulk of the system [13,15] and to provide analytical results which remain finite in the limit of zero wall friction (see Section 2). In the second part of the paper, we explore the validity and robustness of the analytical findings by performing numerical simulations of the flow of concentrated 2d emulsions under the effect of a linear friction.

The paper is organized as follows: in Section 2 we recall the essential features of the theoretical framework for the problem at hand; in Section 3 we derive analytical results for a Bingham fluid; in Section 4 we recall the essential features of the numerical model used to perform the numerical simulations, while in Section 5 we compare the numerical results with the analytical predictions of Section 3. Conclusions and implications for further studies are finally discussed in Section 6.

2. Problem statement

In this section we briefly recall the essential features of the fluid-dynamical model we consider for our study. The model considers a steady 2-dimensional flow in a Hele–Shaw cell with a width *H* and vanishing inertia. We also neglect end effects and assume that the flow is streamwise invariant. Hence, the flow field is unidirectional and writes: $\mathbf{v}=v(z)\hat{x}$, with $\hat{x}$ the streamwise direction and $\hat{z}$ the spanwise one (with $z\in[-H/2\,;+H/2]$), and the problem reduces to a 1-dimensional one for the velocity profile $v(z)$. We set the velocity at the boundaries such that $v(\pm H/2)=\pm v_w$. The bulk wall friction is modelled following Janiaud et al. model [21], i.e. adding a linear friction force $\mathbf{F}_D=-\beta\mathbf{v}$ in the momentum balance equation which, then, reads

$$\frac{d\sigma(z)}{dz}-\beta v(z)=0, \tag{2}$$

with β a wall friction parameter and $\sigma(z)$ the total shear stress. In principle, a more realistic modelling of foams would require a non-linear law for the friction force, i.e. $F_D=-\beta v^{\alpha}$, with $\alpha<1$ [23–25], which would, however, make any analytical approach unfeasible; furthermore, we have recently shown, by comparison with experimental data on Poiseuille flows, that the effect of non-linearities of wall friction on the flow profiles is rather weak [16]. Taking into account *cooperativity* and non-local effects induced by local plastic rearrangements is also key for our purposes. The underlying idea is that correlations among plastic events exhibit a complex spatio-temporal scenario: they are correlated at the microscopic level with a corresponding cooperativity flow behavior at the macroscopic level. Plastic events trigger avalanches of such processes in their vicinity and the consequent non-local effects are captured in terms of the effective inverse viscosity, or fluidity, $f(z)=\dot{\gamma}(z)/\sigma(z)$, relating stress to strain rate $\dot{\gamma}(z)$, locally. At the mathematical level, this is translated in the following equation

$$\xi^2\frac{d^2f(z)}{dz^2}+[f_b(\sigma)-f(z)]=0 \tag{3}$$