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On the capillary pressure function in porous media based on relative permeabilities of two immiscible fluids

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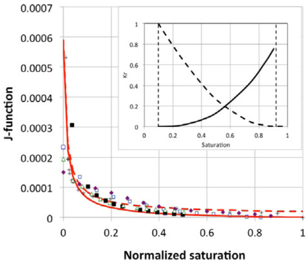
HIGHLIGHTS

- An analytical approach for determination of a capillary pressure (P_c) curve in porous media is proposed.
- P_c is calculated based on wetting and nonwetting phases relative permeabilities and apparent specific surface area.
- The Leverett J_m-function described using the Weibull distribution model.

GRAPHICAL ABSTRACT



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ABSTRACT

The authors propose a new analytical approach and derive an explicit formula to determine a capillary pressure (P_c) curve in porous media, by combining the first principles of surface science with the classical concept of the phase relative permeability. The developed formulae for P_c and a modified Leverett J_m-function are based on the relative permeability functions for the wetting and nonwetting phases, an apparent specific surface area, and an apparent (calculated) contact angle Θ. The application of the proposed approach was tested using several sets of data from existing publications. The developed J_m-function can be described using the Weibull distribution model for drainage and imbibition conditions. The new approach can generally be used for any type of relative permeability functions and different types of wetting and nonwetting phases.

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1. Introduction

Knowledge of reliable capillary pressure vs. saturation and relative permeability vs. saturation relationships is an important aspect in the field of numerical simulations of transport of non-aqueous phase liquids (NAPLs), including hydrocarbon liquids (oils), Dense NAPLs (DNAPLs), and Light NAPLs (LNAPs), in the subsurface. In their recent review of theoretical and experimental studies on the topic of spontaneous imbibition into porous and fractured media, Mason and Morrow [1] noted that there are fundamental problems with the application of differential equations for modeling, because of the nature and choice of appropriate relationships for the two (wetting and nonwetting) relative permeabilities and capillary pressure as functions of saturation. This issue has never been addressed in detail, and it is not clear how these functions should be determined using experiments [1,2]. Several studies showed that the use of a single effective average contact angle, Θ, is not a physically correct approach for systems where there is a distribution of contact angles, especially in the event of the fractional wettability, nonwetting phase entrapment, and redistribution of

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nonwetting phase between different types of pores [3–4,6–9][3–9]. Direct measurement of two-fluid interfacial areas is difficult [8,10].

In this paper, the authors propose to determine first an apparent specific surface area, using an explicit combination of the relative permeability functions for the wetting and nonwetting phases, which then will be used to assess the capillary pressure-saturation function. The results of calculations using a new approach are demonstrated for the drainage and imbibition capillary pressure curves obtained using the data from [11–13].

2. Theoretical background of proposed approach

Numerical simulations of transport phenomena of two immiscible fluids, such as NAPL and water, in porous media are usually based on using the Darcy law given by

$$q_n = -\frac{KK_{rn}}{\mu_n} grad\, P_n$$

$$q_w = -\frac{KK_{rw}}{\mu_w} grad\, P_w$$

where q_n and q_w are the volumetric flow rates of the nonwetting and wetting phases, correspondingly, through the unit area, normal to flow; K is the porous media absolute permeability; K_{rn} and K_{rw} are relative permeability of nonwetting and wetting phases, correspondingly; μ_n and μ_w are viscosity of nonwetting and wetting phases, correspondingly, P_n and P_w are pressures of nonwetting and wetting phases, and capillary pressure is given by

$$P_c = P_n - P_w$$

For the purpose of simulation of flow of two immiscible fluids, P_c is often expressed by means of the Leverett-*J* function [14]. Purcell [15] introduced a relation between the permeability and capillary pressure. In his comments to the Purcell paper, Rose [16] presented an expression for relative permeability, using the fractional wetting phase saturation and Leverett's capillary pressure function. Burdine [17] introduced a tortuosity factor in the model describing the relationship between the relative permeability and capillary function, which for the case of two-phase flow is given by

$$K_{rw} = (\lambda_w)^2 \frac{\int_0^{S_w} \mathrm{d}S_w/P_c^2}{\int_0^1 \mathrm{d}S_w/P_c^2} \qquad K_{rn} = (\lambda_n)^2 \frac{\int_{S_w}^1 \mathrm{d}S_n/P_c^2}{\int_0^1 \mathrm{d}S_n/P_c^2} \tag{1}$$

where λ_w and λ_n are tortuosity factors, and symbols *w* and *n* denote the wetting and nonwetting phases, respectively, and S_w and S_n are saturations of the wetting and nonwetting phases, respectfully. However, the tortuosity factors are not explicitly defined and cannot be measured, thus, they are just fitting parameters.

Rapoport and Leas [18] were probably the first who suggested that the capillary pressure (P_c) and relative permeability functions are dependent on interfacial areas (although they have been typically modeled as functions of fluid saturation). In the past about 20–25 years, various numerical models and modeling techniques have been developed to simulate subsurface multiphase flow at the pore scale, such as pore network models and Lattice-Boltzmann models [19–22]. An application of a thermodynamically constrained macroscale description of flow in porous media, explicitly taking into account the presence of interfaces, was proposed in [23,24]. Despite the wide interest in measuring and calculating specific interfacial area and capillary pressure, there are surprisingly very few works on this subject [25].

Contrary to single-phase flow, when the specific surface area averaged over the total volume of a core sample, is a constant value, in the case of two-phase flow of immiscible fluids, the total specific surface area is a function of the phase distributions, and the total specific surface area is the sum of phase specific surface areas. The wetting phase tends to reside in smaller pores, while nonwetting fluid resides in larger pores. At residual wetting and nonwetting saturations the specific surface values asymptotically approach infinity. Indeed, the residual wetting phase is located in the smallest capillaries, as tiny ganglia, or as pendular drops, bridging the solid grains. All these configurations have very small radii, and, thus, high capillary pressure, exhibiting high capillary resistance to flow, which the pressure gradient, imposed by displacement fluid flow, is incapable to overcome. Complete immobilization of the residual fluid is either equivalent to apparent radii approaching zero value, or to specific surface approaching infinity. In other words, capillarity effects should be taken into account in the evaluation of the specific surface area for a two-phase flow system in porous media, as these effects induce additional resistance to flow. Even in simplified droplet train model, capillary forces are capable of inducing additional resistance to flow [26].

Babchin and Nasr [27] derived a simplified analytical expression for the capillary pressure gradient in homogeneous porous two-fluid media, containing three bulk phases (solid phase, wetting and nonwetting fluids), with three possible interfaces (solid-wetting, solid-nonwetting, and nonwetting-wetting). They assumed a finite value of the three-phase contact angle between two fluids and a solid phase of porous rock, and that the contact surface area between two continuous fluid phases is small in comparison with contact surface areas between each fluid and porous rock in the unit volume of the rock. The surface energy of such a system can be given by

$$W = \gamma_{n-s} \cdot S_v \cdot S_n + \gamma_{w-s} \cdot S_v \cdot S_w + \gamma_{n-w}\sigma_{n-w}, \tag{2}$$

where W is the density of excess energy of porous media associated with interfaces, and in other words, it is the additional density of surface energy, due to the presence of two fluids within porous rock, S_v is the specific surface area of porous media, $\gamma_{n\text{-}s}$, $\gamma_{w\text{-}s}$ and $\gamma_{n\text{-}w}$ are nonwetting phase-solid, wetting phase-solid, and nonwetting–wetting phases specific surface energies, respectively, S_n and S_w are saturations of nonwetting liquid and water, subject to $S_n + S_w = 1$, and $\sigma_{n\text{-}w}$ is area of nonwetting-wetting phases interface within unit volume of porous media.

According to [27], the area of direct contact between two immiscible fluids in a one-Darcy permeability porous media is five orders of magnitude smaller than the area of contact between these fluids and solid grains, therefore the last term of Eq. (2) can be neglected. (For solid–liquid and liquid–liquid contact areas to be of the same order, one fluid should be emulsified in another, and droplet size should be of the same order of magnitude as the solid grain size. Such a state is known as emulsion flow, when one of the fluids is in discontinuous state, and no three phase contact angle exist. This kind of flow is rarely observed and is outside the scope of this paper.)

Equation (2) can then be given by

$$W \approx (\gamma_{ns} - \gamma_{w-s}) \cdot S_v \cdot S_n + \gamma_{w-s} \cdot S_v \tag{3}$$

Using the standard Young equation, $\gamma_{n\text{-}s} - \gamma_{w\text{-}s} = \gamma_{n\text{-}w} \cos \Theta$, where Θ is wetting-phase contact angle, we can express W as

$$W \approx \gamma_{n\text{-}w} \cdot \cos \Theta \cdot S_v \cdot S_n + \gamma_{w\text{-}s} \cdot S_v \tag{4}$$

The second term in Equation (4) corresponds to the condition of the full water saturation of the porous space, when $S_w = 1$. The first term of Eq. (4) represents the energy arising from the capillary pressure, which is developed in the system of two immiscible fluids in porous media.

$$P_c \approx \gamma_{n\text{-}w} \cdot \cos \Theta \cdot S_v \cdot S_n \tag{5}$$