Colloids and Surfaces A: Physicochem. Eng. Aspects 382 (2011) 24–31

Contents lists available at ScienceDirect

Colloids and Surfaces A: Physicochemical and Engineering Aspects

journal homepage: www.elsevier.com/locate/colsurfa

Ordered polyhedral foams in tubes with circular, triangular and square cross-section

S.T. Tobin*, J.D. Barry, A.J. Meagher, B. Bulfin, C.E. O'Rathaille, S. Hutzler

School of Physics, Trinity College Dublin, Dublin 2, Ireland

ARTICLE INFO

Article history:
Received 5 October 2010
Received in revised form 4 November 2010
Accepted 9 November 2010
Available online 18 November 2010

Keywords:
Ordered structure
Foam structure
Foam modelling

ABSTRACT

Soap bubbles of equal volume readily crystallize as ordered polyhedral foam structures when introduced into tubes whose width is of the same order as the bubble diameter. In the past a large number of these structures have been identified experimentally for cylindrical tubes. The surface energy per bubble was computed using Ken Brakke's Surface Evolver software. We have now extended this work to tubes with square and triangular cross-sections and present both experimental data and results of Surface Evolver calculations for a variety of new ordered foam structures. The result is a catalogue of all structures for which simulation results exist.

1. Introduction

When soap bubbles of equal volume are introduced into tubes they readily crystallize as ordered polyhedral foam structures, provided that the tube diameter is of the same order as the bubble diameter (and larger than the capillary length, in order to form dry foams [1]). This phenomenon was noted as early as 1933 by Mann and Stephens [2], who reported three ordered structures in tubes of cylindrical cross-section.

In recent years monodisperse foams have enjoyed renewed interest as objects of study [3], with a large number of ordered monodisperse structures identified experimentally in *cylindrical* tubes [4,5,12] for foams (and emulsions [6]). Several structures have also been identified in the *square* geometry [7]. While in the simplest of these structures all bubbles are in contact with the tube wall, several contain at least one bulk bubble per unit cell.

The goal of this article is to provide a comprehensive catalogue of the simplest structures for which simulation results exist. We also present new structures from experiment and simulation for tubes with *triangular* and *square* cross-sections.

Such ordered foam structures may have direct applications in discrete microfluidics and lab-on-a-chip technologies. Microfluidics is the science and technology of systems that process or manipulate small amounts of fluids using channels with dimensions of tens to hundreds of micrometers [8]. *Discrete* microfluidic systems employ droplets, bubbles, or foams [9–11], with applications on the larger-than-micron scale. Studying the structure and rheology of confined foams (and emulsions) will be key to designing such systems.

2. The study of ordered foams in confinement

2.1. Experimental methods

An experimental set-up that may be conveniently used to produce ordered foam structures is sketched in Fig. 1. A glass or perspex tube of diameter 1–2 cm is partly submerged in a surfactant solution (commercial detergents which produce very stable foams are sufficient for these types of experiments which are concerned with foam structure only). Air or nitrogen gas is injected into the solution at the submerged end of the tube (through a capillary). Monodispersity of bubbles is achieved using a pump or gas storage tank to ensure constant gas pressure. The bubble size is controlled via the diameter of the capillary opening and the gas pressure. As the bubbles are collected in the tube, they readily self-assemble into ordered structures, such as that shown in Fig. 2.

The type of structure formed as the bubbles fill the tube is crucially dependent on the ratio λ of the tube diameter to the bubble diameter. In the earliest comprehensive study conducted, Weaire et al. [12] identified 11 structures using a setup of the type shown in Fig. 1. Pittet et al. [4] extended the catalogue of cylindrical structures to 37 and included the experimentally determined ranges of λ for 11 of these. However, only nine of these structures have ever been simulated [5,13].

The experimental determination of bubble volume and of λ is as follows:

* Corresponding author. Tel.: +353 1 896 1469.
E-mail address: tobinst@tcd.ie (S.T. Tobin).

0927-7757/$ – see front matter © 2010 Elsevier B.V. All rights reserved.
doi:10.1016/j.colsurfa.2010.11.024

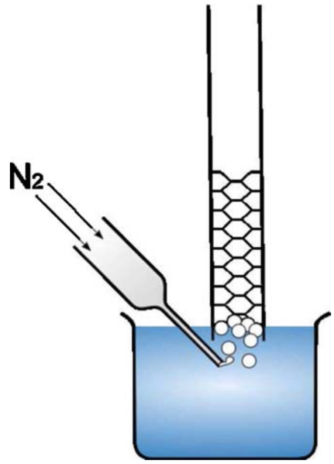

Fig. 1. A schematic representation of the apparatus used to produce ordered foam structures. Equal bubble volume, achieved by having constant air-flow, is the most important factor for obtaining ordered foam structures.

1. A long column of the desired structure is allowed to form. The length l of a section of the tube that contains a multiple of the unit cell (i.e., a simple periodic subset of the structure) of the ordered structure is measured. The volume of this section is therefore

$$V_{\text{tot}} = A_{\text{cs}} l \quad (1)$$

where A_{cs} is the cross-sectional area of the tube.

2. The volume of a single bubble V_b is then calculated by dividing V_{tot} by the product of the number of bubbles in a unit cell of the structure and the number of unit cells in the section. The equivalent sphere radius of a bubble is then given by:

$$r = \left(\frac{3V_b}{4\pi}\right)^{1/3} \quad (2)$$

3. The ratio λ of characteristic tube width to bubble radius is then calculated, based on the definitions displayed in Fig. 3.

As can be seen from the definition of λ, it may be varied by either varying the tube diameter (by choosing a different tube) or by varying the bubble diameter. Varying the latter allows for smooth changes, as the bubble volume is easily controllable by the adjustments of the gas pressure.

An additional adjustment mechanism is available when studying ordered foams using ferrofluids (to which surfactant is added) [14,15]. In this case, bubble volume may be controlled by applying a magnetic field gradient close to the nozzle and varying the field strength, to change the effective buoyancy of the emerging bubbles.

Yet another experimental method (which would deserve to be further investigated) for adjusting bubble volume is developed by Boltenhagen et al. [16]. An ordered foam structure was created in a cylindrical tube, consisting of a fixed number of bubbles N. The structure is confined between a stopper and a movable piston. Moving the piston allows the volume of the bubbles to be changed continuously. By compressing and dilating (up to 500% of initial volume) the foam, structural transitions may be induced.

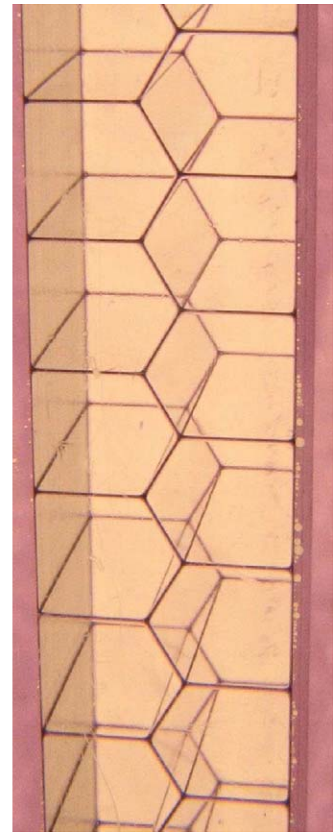

Fig. 2. An example of an ordered foam structure in a tube with square cross-section (side length 1 cm). The structure has two bubbles per unit cell and is recorded as s-2.

Determining precise values for the endpoints of the ranges of λ over which particular structures can exist is complicated by the effect of templating. In all the experiments mentioned above, some foam of an earlier λ value was already in the tube as the value of λ was changed. Bubbles may thus be forced into a 'wrong' structure. This holds both for the case where λ is changed by compression, or when bubbles with a different V_b value are introduced (this is similar to a lattice mismatch in crystallography).

2.2. Simulation methods

The question arising from confined foam experiments is: for some given λ value, what determines which ordered structure forms, or which alternatives are available? It is hypothesised that the structure that forms is generally the structure with least sur-

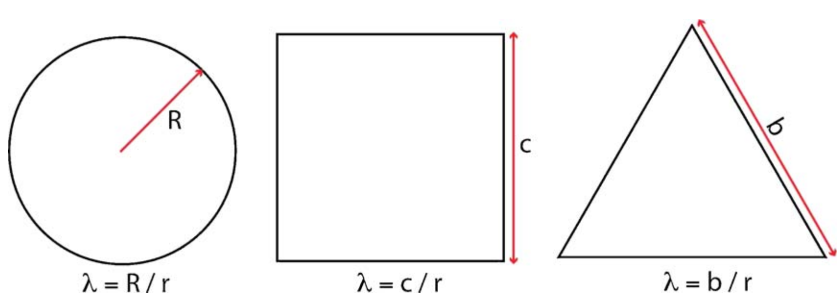

Fig. 3. Definitions of λ for various tube cross-sections. r is the equivalent sphere radius of a bubble of volume V_b, defined in Eq. (2).