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ScienceDirect
Colloids and Surfaces A: Physicochem. Eng. Aspects 309 (2007) 20–29
COLLOIDS AND SURFACES A
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A single sagging Plateau border

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Received 27 July 2006; received in revised form 3 November 2006; accepted 7 November 2006
Available online 15 November 2006

Abstract

The loading of foams with liquid weight, contributed primarily by the Plateau borders, results in an external force which may often be important to structure and drainage, especially in foams of a higher liquid content; loading induces deformations in structures. A Plateau border in a three-dimensional foam is supported by three films, which are assumed to sag under loading, initially with linear elasticity in these numerical simulations. The weight of a Plateau border is allowed to vary along its length, in accordance with the foam drainage equation for a single channel (balancing gravity and capillarity for a given channel size and orientation). The surface area of the deformed structure is subsequently minimized in the Surface Evolver, and it is found that only for sufficiently dry borders, the films have linear elasticity. For wet foams, the true distortion due to loading is larger than the linear model predicts; an iterative approach, based on the solution of the drainage equation and a direct application of gravitational forces in the Surface Evolver, is developed for wetter Plateau borders. The implications for forced-drainage experiments and continuum-level drainage models are discussed.

Keywords: Foam drainage; Foam structure; Non-linear elasticity; Surface Evolver

1. Introduction

A stationary liquid foam is usually treated as a collection of polyhedral gas bubbles separated by a finite amount of liquid, which is contained predominantly in the Plateau border channels that would form the edges of surfaces in a corresponding perfectly dry polyhedral structure [1,2]. The gravitational weight of these channels is neglected in standard theories, except those in which it is required as a driving force for the drainage of liquid through a foam [3–5]. This is not, of course, a strictly accurate picture; the liquid has weight which acts on the films (faces of the bubble polyhedra), thereby affecting the foam structure.

Some work has been done by Weaire et al. [6] on a uniformly loaded, two-dimensional honeycomb foam. The authors found, as a result of their simulations, that a loaded foam has slightly different structure than is assumed by the standard theories. For instance, with an external load applied, there can exist gas pressure gradients in a monodisperse foam (in 2D, this is normally only possible as a result of polydispersity). In addition, the symmetry of vertices in the foam could be disturbed by loading, requiring modifications to existing theories (such as the continuum foam drainage equation) which use the assumption of randomly oriented structural elements (e.g. borders).

This work in two dimensions has provided many useful observations. However, the sagging of real liquid channels is not caused by the application of a uniform load. Gravity acts vertically, and the component normal to the film is the force that results in distortion of the regular structure; since the film surface will curve due to the application of an external force, the load is therefore neither uniform in direction, nor in magnitude.

Real Plateau borders also present a 3D problem, not a 2D one. The flow through the channels (and thus the channel cross-sectional area and weight) is determined by the drainage equation for a single Plateau border. A 2D loaded foam model does not account for this channel drainage flow rate, nor does it explicitly acknowledge that it must be uniform along a single channel. Such factors, affecting the gravitational loading itself, cause the channel cross-section to distort normal to its length and by differing amounts along its length. Thus the problem becomes inherently three-dimensional. It is known that external influences (e.g. imposed shear) on a foam sample affect its structure and thereby drainage properties [7]; there is every reason to

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doi:10.1016/j.colsurfa.2006.11.016

believe that internal loading forces could have similar effect.

To analyse the effect of gravitational loading on 3D foam structures, this paper takes a single channel as an example. In the following section we present a model for a single sagging Plateau border. The model initially uses the assumption of linear elasticity of the deformations—which is not, of course, a valid assumption for real foam films. Following the presentation of the model, we give results of our numerical simulations. We then test the validity of the linear model in the Surface Evolver. After that we describe Evolver simulations for non-linear elasticity, and in the last section we present conclusions.

2. Linear elasticity model

To model the effect of gravitational loading of Plateau borders (only, as opposed to on vertices) on bubbles in a foam, a simple model is developed in which the borders are held up by surface tension (i.e. there are no volume or pressure constraints on individual bubbles). It is also assumed in the first instance that the deformations are elastic; that is, there is a continuum spring constant measuring the force per unit displacement per undistorted length of the channel.

2.1. Deformation due to loading

In this model, three films which are being distorted by gravitational forces belong to a single Plateau border. One film is assumed vertical, and the other two hang downwards (as presented in Fig. 1(a)). Of these three films, the upper surface will show a gain in energy (or surface area) under sag exactly the same, at leading order, as that gained by the downward-hanging films. It is the second-order effect which is calculated here.

We define a coordinate system such that an undeformed film is in the xy plane. Here x measures a distance transverse to a Plateau border, y a distance along it. Moreover z is a distance normal to the undeformed film (see Fig. 1(b)). For a semi-infinite strip of film, initially on $x \geq 0$ and $0 \leq y \leq L$, the edge of which is distorted by a distance $\epsilon L\ \sin(\pi y/L)$ at an angle $\pi/3$ to the plane of the film, the distorted edge lies along $x = (1/2)\epsilon L\ \sin(\pi y/L)$ and $z = (\sqrt{3}/2)\epsilon L\ \sin(\pi y/L)$ (Note that ϵ determines the amplitude ϵL of the distortion. The vertices at the end points of the Plateau border are assumed to be pinned and the films must stretch to accommodate the effect of liquid weight). The solution for the film shape is $z = \psi(x, y)$. As mentioned previously, we consider an open system with no volume constraints (hence films are surfaces of zero mean curvature). At leading order $\nabla^2\psi(x, y) = 0$, and the boundary condition can be applied along $x = 0$ instead of along the true boundary. Then

$$\psi = \frac{\sqrt{3}}{2}\epsilon L\ \exp\left(-\frac{\pi x}{L}\right)\ \sin\frac{\pi y}{L}. \quad (1)$$

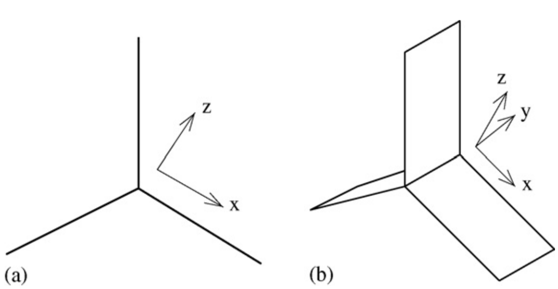

Fig. 1. (a and b) Coordinate system for the derivation of the linear elasticity model. The orientation assumed is that the upper film is in the vertical plane with two downward-hanging films coming off it. Coordinates x, y, and z are defined such that for one of the two downward-hanging films x is along the film, y along the Plateau border, and z normal to the undistorted film. The coordinate y is along the border but not necessarily horizontal, since the border can be tilted such that one vertex lies lower than the other, allowing for drainage along it. Part (a) of the figure is a projection of part (b) into the xz plane.

2.1.1. Area and energy increase due to distortion

An element $\mathrm{d}x, \mathrm{d}y$ is distorted to $\mathrm{d}x(1, 0, \partial\psi/\partial x)$ and $\mathrm{d}y(0, 1, \partial\psi/\partial y)$, the area of which, S, is found by the magnitude of the cross product vector to be

$$\mathrm{d}S = \mathrm{d}x\,\mathrm{d}y\left(1 + \frac{1}{2}\left(\frac{\partial\psi}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial\psi}{\partial y}\right)^2\right)$$
$$= \mathrm{d}x\,\mathrm{d}y\left(1 + \frac{3}{4}\epsilon^2\pi^2\ \exp\left(-\frac{2\pi x}{L}\right)\right). \quad (2)$$

Integrating this over the *original* domain gives

$$S = S_{\text{original}} + \frac{3\pi}{8}\epsilon^2 L^2, \quad (3)$$

where S_{original} is the area S for the undistorted structure.

There is also an area variation arising from the fact that the domain has been shifted from $x = 0$ to $x = (1/2)\epsilon L\ \sin(\pi y/L)$. This can be obtained by projecting the shift onto the $z = 0$ plane. Alternatively, if the distorted film belongs to a Plateau border with one film upwards and two hanging diagonally downwards, the energy loss from the new domain in the two diagonal films (if the Plateau border is distorted downwards) exactly matches the energy gain from the vertical film, which stays flat but increases in surface area. Thus, the total area gain in the two diagonal films is $(3\pi/4)\epsilon^2 L^2$ and the total energy gain is $(3\pi/2)\epsilon^2 L^2\sigma$ (where σ is surface tension).

This all assumes that the half-period of the disturbance is simply some length L. It would be useful to generalize this to a length L/n and generalize ϵ to $n\epsilon$ (to keep the disturbance amplitude ϵL the same). As a result, the energy increase for each L/n segment is kept the same, and the total energy increase over all n segments is $(3\pi n/2)\epsilon^2 L^2\sigma$. This energy increase can be used to estimate an effective spring constant for each mode of disturbance. Note that this will be a *continuum* spring constant: it measures the force per displacement per unit original length of Plateau border.

There is one subtlety involved with relating displacement to energy increase: namely that displacement (which we denote $\mathcal{X}$) varies like $\sin\ \pi n y/L$ along the mode. A displacement amplitude ϵL implies a root mean square displacement $\mathcal{X}_{\text{rms}} = \epsilon L/\sqrt{2}$. An rms displacement is an appropriate measure, because energy is assumed proportional to the square of displacement which varies spatially like $\sin^2(\pi n y/L)$ along the mode: the root mean square averages over this variation. Thus mode energy is $3\pi n\mathcal{X}_{\text{rms}}^2\sigma$