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Neural representations of magnitude for natural and rational numbers

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ABSTRACT

Humans have developed multiple symbolic representations for numbers, including natural numbers (positive integers) as well as rational numbers (both fractions and decimals). Despite a considerable body of behavioral and neuroimaging research, it is currently unknown whether different notations map onto a single, fully abstract, magnitude code, or whether separate representations exist for specific number types (e.g., natural versus rational) or number representations (e.g., base-10 versus fractions). We address this question by comparing brain metabolic response during a magnitude comparison task involving (on different trials) integers, decimals, and fractions. Univariate and multivariate analyses revealed that the strength and pattern of activation for fractions differed systematically, within the intraparietal sulcus, from that of both decimals and integers, while the latter two number representations appeared virtually indistinguishable. These results demonstrate that the two major notations formats for rational numbers, fractions and decimals, evoke distinct neural representations of magnitude, with decimals representations being more closely linked to those of integers than to those of magnitude-equivalent fractions. Our findings thus suggest that number representation (base-10 versus fractions) is an important organizational principle for the neural substrate underlying mathematical cognition.

Introduction

Representations of symbolic number types

Humans are unique in having developed symbolic notations for numbers. Given that a primary function of numbers is to convey magnitude values, it is important to understand the mental and neural representations of numerical magnitudes. The goal of the current study was to address the question of how different symbolic notations (natural numbers, fractions, and decimals) map onto magnitude codes. Specifically, we sought to determine whether different notations map onto a single, fully abstract, magnitude code, or whether separate representations exist for specific number types (e.g., natural versus rational) or number representations (e.g., base-10 versus fractions).

Numerous studies of numerical magnitude comparisons have yielded a *symbolic distance effect*: comparisons of numbers that are closer in magnitude (e.g., 7 vs. 8) are slower and more error prone than comparisons of numbers that are farther apart (e.g., 2 vs. 8; Moyer and Landauer, 1967; Holyoak, 1978). A similar distance effect is observed in children (Barth et al., 2005; Brannon, 2002). Rhesus monkeys display a distance effect for numerosity comparisons; moreover, they are capable of learning shapes (Arabic numerals) corresponding to small numerosities (1–4 dots), such that the shapes acquire neural representations overlapping those of the corresponding perceptual numerosities (Diester and Nieder, 2007).

The distance effect and other phenomena have been interpreted as indications that numerical magnitudes (at least for integers) are associated with an analog magnitude representation akin to a mental number line (Dehaene and Changeux, 1993; Gallistel, 1993; Opfer and Siegler, 2012). Neuroimaging studies with both adults and children have implicated the intraparietal sulcus (IPS) as the central area for representing and comparing symbolic integer magnitudes (and also non-symbolic magnitudes) (Dehaene et al., 2003; Nieder and Dehaene, 2009; Piazza et al., 2007; Pinel et al., 2001). Further, IPS activation is inversely related to the numerical distance between two numbers being compared (Cohen Kadosh et al., 2005; Kaufmann et al., 2005), consistent with the behavioral distance effect.

While the representation of whole-number magnitude has received considerable attention, far less is known about the representation of other symbolic number types, such as the rational numbers (fractions and decimals). Some have argued that the representation of magnitude in general is entirely abstract, and that all symbolic and non-symbolic magnitudes can be represented using a single mental (and neural) number line (Eger et al., 2003; Naccache and Dehaene, 2001; Siegler

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et al., 2011). However, studies investigating this topic have as yet failed to reach a consensus. Previous behavioral research has mainly focused on the extent to which fractions are represented holistically. This work has focused on the issue of whether the overall (holistic) magnitude of a fraction is accessed automatically, like an integer (Kallai and Tzelgov, 2009; Meert et al., 2010a, 2010b; Schneider and Siegler, 2010; Sprute and Temple, 2011). Evidence for holistic magnitude representation come from studies examining the distance effect during fraction comparisons. Many studies (e.g. Schneider and Siegler, 2010) have found that adults show a distance effect when representing fractions during comparisons. However, other studies have shown that depending on the stimuli and availability of various shortcut strategies, adults may represent only the whole-number components of the fraction and not its holistic magnitude (e.g. Bonato et al., 2007; Fazio et al., 2015).

Moreover, other work has shown that even when a distance effect is found for fraction comparisons, the size and scale of the effect is entirely different for fractions relative to either integers or decimals. DeWolf et al. (2014) had adults compare fractions, matched decimals (rounded to three digits) and integers (created by multiplying the equivalent decimal by 1000 to obtain a three-digit integer). Comparisons for all three number types yielded reliable distance effects, based on the holistic magnitudes of the numbers being compared. Importantly, however, response times and error rates for the fraction comparisons were much higher than for comparisons of either decimals or integers, with the latter number types showing no differences in response times or errors. Moreover, the distance effect was much more pronounced for fractions, with response times averaging between 2 and 8 s for far versus near number pairs. In contrast, response times for integers and decimals overlapped with one another, and generally were no longer than 2 s. This dramatic difference in the scale of the distance effect across number types suggests that the magnitude information associated with fractions may be less precise than that associated with integers or decimals, and that the process of accessing magnitudes is more effortful and less automatic for fractions than for either integer or decimal formats.

Using fMRI to investigate magnitude representation

Behavioral research investigating rational number magnitudes suggests there are important differences between magnitude processing for fractions relative to other number types. Although neuroimaging methods, and functional magnetic resonance imaging (fMRI) in particular, have been employed to assess the neural substrates of numerical magnitude representation (e.g., Darmla and Just, 2013), numerical symbols representations (see Ansari, 2016) and algebra (e.g., Monti et al., 2012), there is no consensus regarding the interpretation of the behavioral differences observed between fractions and other number types. The present study applied neuroimaging methods to assess the relationships among the neural representations of magnitude for different symbolic formats. If the representation of magnitude is entirely abstract, then the neural representations of a fraction and its magnitude-equivalent decimal (e.g., 2/5 vs. 0.40) in the IPS might be expected to be identical. In contrast, if fractions and decimals are processed very differently (as some behavioral studies suggest), then the neural codes for the different notations may differ. To date, these alternative predictions remain untested. In fact, only two studies have ever probed the neural representations underlying the processing of fractional numbers (Ishebeck et al., 2009; Jacob and Nieder, 2009a), and neither of these assessed the neural representations underlying decimal numbers, or the relationship between neural representations of magnitude across different formats for rational numbers.

A few other studies have examined how neural representations of magnitude differ as a function of notation by comparing neural responses to whole numbers versus their verbal equivalents (e.g., "12" versus "twelve"). Some studies have found that IPS activation was notation-independent (Eger et al., 2003; Naccache and Dehaene, 2001), whereas other studies suggest there may be both notation-specific and notation-independent areas (Bluthe et al., 2015; Cohen Kadosh et al., 2007; Darmla and Just, 2013). However, these studies all compared a single mathematical notation (whole numbers) versus natural language (number names). No work has been done to investigate the question of whether alternative mathematical formats, such as fractions versus decimals, evoke similar or distinct neural representations of magnitude.

As noted above, only two studies have investigated the representation of symbolic fraction magnitudes using fMRI. Jacob and Nieder (2009a) used an adaptation paradigm to test symbolic fraction magnitudes (single and multi-digit fractions). Recovery in the BOLD signal after habituation was observed in the frontoparietal cortex, and specifically the IPS. The pattern of signal recovery was the same after presentation of either a new symbolic fraction (e.g., "1/2") or a new fraction written as a word (e.g., "half"), suggesting that fractions and their verbal equivalents recruit the same or overlapping neural areas.

The second study that investigated symbolic fraction notation with fMRI used a magnitude comparison paradigm, rather than an adaptation paradigm. Ishebeck et al. (2009) had adult participants perform a simple magnitude comparison task with fractions, in which participants saw two fractions simultaneously on the screen and pressed a button to indicate which was larger in numerical magnitude. The stimuli included different types of fraction pairs, some with common components, in order to enable a variety of potential strategies during the comparison process. The results showed that activity in the right IPS was inversely correlated with the distance between the two fractions based on their holistic magnitude difference, and not with the distances between any component parts. Ischebeck et al. interpreted their fMRI results as supporting the hypothesis that (despite an opportunity to use componential strategies) fraction comparisons were performed using holistic magnitudes.

However, neither Ishebeck et al. (2009) nor Jacob and Nieder (2009a) directly compared processing of fractions with that of other symbolic formats. Although previous work indicates that magnitude representations for fractions involve roughly the same general neural area (the IPS) as do magnitude representations for symbolic integers (and non-symbolic numerosities; see Jacob and Nieder, 2009b; Jacob et al., 2012), the extent to which processing and representation of magnitude is the same or different for fractions relative to other number types has not been examined. Furthermore, the more general question of whether different symbolic formats for numbers evoke the same or different abstract magnitude representations remains unanswered.

The present study

In the present experiment, we employ univariate and multivariate analysis of fMRI data to compare, in a within-subject design, the neural representations of magnitude across different symbolic notations (integers, decimals, and fractions). We hypothesized that, consistent with previous research, all of the number types would activate the IPS. The main questions concerned possible differences between the number types. If all number types activate the same abstract neural representation (based on relative rather than absolute magnitude, to take account of the scale difference between integers and rational numbers), then no differences among the number types would be expected. A second possibility is that neural activation of integers will differ from that of rational numbers (either fractions or decimals), both because the latter are more complex and because the overall magnitude scale differs. A third possibility, based on the behavioral findings of DeWolf et al. (2014), is that fractions will evoke a neural signature distinct from that of either magnitude-equivalent decimals or integers, whereas the latter two number types will evoke similar activation patterns.