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Bayesian electromagnetic spatio-temporal imaging of extended sources with Markov Random Field and temporal basis expansion

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ABSTRACT

Estimating the locations and spatial extents of brain sources poses a long-standing challenge for electroencephalography and magnetoencephalography (E/MEG) source imaging. In the present work, a novel source imaging method, Bayesian Electromagnetic Spatio-Temporal Imaging of Extended Sources (BESTIES), which is built upon a Bayesian framework that determines the spatio-temporal smoothness of source activities in a fully data-driven fashion, is proposed to address this challenge. In particular, a Markov Random Field (MRF), which can precisely capture local cortical interactions, is employed to characterize the spatial smoothness of source activities, the temporal dynamics of which are modeled by a set of temporal basis functions (TBFs). Crucially, all of the unknowns in the MRF and TBF models are learned from the data. To accomplish model inference efficiently on high-resolution source spaces, a scalable algorithm is developed to approximate the posterior distribution of the source activities, which is based on the variational Bayesian inference and convex analysis. The performance of BESTIES is assessed using both simulated and actual human E/MEG data. Compared with L_2-norm constrained methods, BESTIES is superior in reconstructing extended sources with less spatial diffusion and less localization error. By virtue of the MRF, BESTIES also overcomes the drawback of over-focal estimates in sparse constrained methods.

1. Introduction

Reconstructing brain activities from non-invasive electroencephalography (EEG) or magnetoencephalography (MEG) measurements, also known as E/MEG source imaging, is a long-standing inverse problem that is intrinsically and highly ill-posed: the solution will be non-unique without effective anatomical or functional constraints restricting the solution space (He et al., 2011; Baillet et al., 2001; Wu et al., 2016). At present, there are two general methodological categories for solving the E/MEG inverse problem. The first is the dipole fitting method (He et al., 1987; Mosher et al., 1992), which attempts to estimate brain activities using a small number of equivalent current dipoles (ECDs). Although this method yields good estimates when the number of active regions and their areas are small, for more complicated source configurations it is less successful. Empirical and theoretical evidence indicates that a major portion of E/MEG signals arises from cortical gray matter, with considerable spatial extents (Hämäläinen et al., 1993; Tao et al., 2005). In the context of epilepsy, estimating the extents of active regions is greatly important for surgical planning. Dipole fitting may locate the center of mass of a cortical patch well, but it is incapable of determining the extent of the patch (He et al., 2011; Ding and He, 2008). Indeed, even optimal ECD estimates may be meaningless in the case of a highly extended cortical area.

The other class of inverse solvers is the distributed source imaging method, which estimates the amplitudes of a predefined dense set of dipoles that may encompass an entire cortical surface (or may include subcortical regions). Distributed source estimates are typically obtained by solving a linear inverse problem. However, because the dimension of the source space largely outnumbers the dimension of the sensor space (about hundreds of times as the number of sensors), prior constraints are essential to yield a unique solution. The most common constraint is to assume that the underlying sources possess a small overall energy (i.e., L_2-norm), which is the central idea of the minimum norm estimate (MNE) (Hämäläinen and Ilmoniemi, 1994) and its variants (e.g., weighted

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MNE (wMNE) (Dale and Sereno, 1993), standardized low resolution brain electromagnetic tomography (sLORETA) (Pascual-Marqui, 2002), and LORETA (Pascual-Marqui et al., 1994). Despite the low computational requirements of L_2-norm based methods, these methods suffer dramatically from their poor spatial specificities: their estimates tend to spread over multiple cortical sulci and gyri even though the spatial extents of the underlying sources may be small. Moreover, these methods typically underestimate deep sources in favor of more superficial ones. These issues arise because without model selection, L_2-norm based methods always assume a full model that includes the activities of all estimated sources. One type of solution is to use sparse constrained methods, including the L_0-norm constraint (Xu et al., 2010), minimum L_p-norm ($p \le 1$) methods (Ding and He, 2008; Xu et al., 2007), and sparsity-inducing norm afforded by empirical Bayes (also known as sparse Bayesian learning (SBL) (Wipf and Nagarajan, 2009). Nevertheless, as previously reported (He et al., 2011), sparse constrained methods may lead to over-focal estimates, which contain activations at considerably fewer cortical regions than actual sources. Moreover, the reconstructed source activities are often highly discontinuous over time due to noise sensitivity. Thus, both the L_2-norm based methods and sparse constrained methods are limited in accurately estimating the spatio-temporal patterns of extended sources.

Notably, the aforementioned source imaging methods make the assumption of temporal independence (i.e., the source estimation is applied separately to each time point), which ignores the temporal correlation structures that are clearly present in E/MEG measurements. Naturally, the information afforded by such structures could be employed to further regularize the solution space and could thus lead to performance improvement. To date, three strategies have been proposed to utilize temporal information to improve source imaging solutions. The first strategy is to directly incorporate a temporal constraint as a regularization term in optimization problem formulation. For instance, the difference between neighboring time points is penalized in Baillet and Garnero (1997), while the work by Daunizeau et al. (2006) models the temporal smoothness of current sources based on their second derivatives. The second strategy uses state-space models to account for temporal information. For instance, a state-space model is proposed to describe the spatiotemporal correlation of the sources (Galka et al., 2004; Lamus et al., 2012; Yamashita et al., 2004; Liu et al., 2015), and model estimation is performed by using either Kalman filtering (Galka et al., 2004; Lamus et al., 2012) or a recursive penalized least squares procedure (Yamashita et al., 2004; Liu et al., 2015). However, these algorithms are too computationally demanding to be feasible for estimation in a high-resolution source space. Moreover, the underlying autoregressive (AR) model can characterize a stochastic process well but may be less powerful in modeling some specific waveforms, such as the event related potentials (ERPs). The third strategy utilizes temporal basis functions (TBFs) to make use of temporal information. The TBFs are either predefined or estimated in a data-driven manner using E/MEG spatiotemporal decomposition methods. In Gramfort et al. (2013), current sources are represented as a linear combination of Gabor atoms, yielding spatially sparse and temporally smooth estimates via an efficient solver based on proximal iterations by incorporating the time-frequency mixed-norms constraint. The work of Bolstad et al. (2009) seeks estimates comprising a small number of space-time events out of a large number of candidates using the expectation-maximization (EM) algorithm. In Huang et al. (2006), sample-wise L_1-norm estimates are projected into a signal subspace defined by a set of TBFs, achieving improvement over conventional L_1-norm estimates. Similarly, the algorithm in Ou et al. (2009) projects both E/MEG data and current sources onto a set of TBFs and imposes L_1-norm regularization in the space domain and L_2-norm regularization in the temporal domain. There are also methods that model the source activity as the sum of a linear combination of several TBFs (Trujillo-Barreto et al., 2008) or spatiotemporal basis functions (Stahlhut et al., 2013), and a temporal error term. However, spacewise, these approaches yield either over-focal (Gramfort et al., 2013; Bolstad et al., 2009; Huang et al., 2006; Ou et al., 2009) or diffused estimates (Trujillo-Barreto et al., 2008) in reconstruction of extended sources.

Motivated by previous studies (Daunizeau et al., 2006; Gramfort et al., 2013; Trujillo-Barreto et al., 2008; Stahlhut et al., 2013), this paper presents a novel algorithm based on a Bayesian probabilistic model that comprehensively characterizes E/MEG data by exploiting its spatio-temporal structures in a data-driven fashion, with the goal of accurately reconstructing extended sources in both space and time. More specifically, the contributions of the paper are threefold:

1) A flexible Bayesian model is proposed to model the spatio-temporal structure within the E/MEG data. Specifically, Markov Random Field (MRF) is employed to embody spatially contiguous and locally homogeneous brain activities, the temporal smoothness of which is modeled by a set of TBFs derived from spatio-temporal decompositions of E/MEG data.
2) A desirable property of the proposed model is that it encourages sparsity, not on an individual source but rather on a local patch basis, thereby overcoming the over-focal drawback of conventional sparse estimation algorithms.
3) A fully data-driven and computationally efficient algorithm is developed for the inference of the Bayesian model. To enable scalability to high-resolution source spaces, we combine variational Bayes (VB) and empirical Bayes to solve the inverse problem. In particular, VB is employed to compute the variational posterior distribution of the model parameters, and a dual-form representation of free energy is developed to estimate the hyperparameter values, leading to a tractable modified cost function. By optimizing this modified objective function, we obtain a convenient update rule and ensure that free energy increases at each iteration.

1.1. Organization of the paper

The current paper is organized as follows. In Section 2, we introduce the Bayesian spatio-temporal forward model, the associated source imaging algorithm, and the practical implementation of the proposed algorithm. In Section 3, we present the simulation protocol and performance metrics. In Section 4, the proposed method is applied to analyze simulated and human E/MEG data. Section 5 concludes the paper with a discussion of our findings.

1.2. Notation

For notation, bold and roman lowercase variables denote vectors and scalars, respectively. Bold uppercase variables denote matrices. The transpose of a matrix $\boldsymbol{X}$ is denoted by $\boldsymbol{X}^{\top}$. We also use x_j^i to denote the $(i,j)^{\text{th}}$ element of matrix $\boldsymbol{X}$, and $\boldsymbol{x}_j$ to denote the j^{th} column of matrix $\boldsymbol{X}$. $\mathcal{N}(\boldsymbol{x}|\mu, \boldsymbol{\Sigma})$ is a multivariate Gaussian density over $\boldsymbol{x}$, with a mean μ and a covariance $\boldsymbol{\Sigma}$. $\text{tr}(\boldsymbol{X})$ denotes the trace of the matrix $\boldsymbol{X}$.

2. Generative model

2.1. Probabilistic spatio-temporal data model

Based on the quasi-static approximation of Maxwell's equations, the observed E/MEG signals are a linear function of brain current sources

$$\boldsymbol{B} = \boldsymbol{L}\boldsymbol{S} + \varepsilon \tag{1}$$

where $\boldsymbol{B} = [\boldsymbol{b}_1, \boldsymbol{b}_2, \cdots, \boldsymbol{b}_T] \in \mathbb{R}^{P \times T}$ is the measured E/MEG signal, P is the number of sensors, and T is the number of time points at which measurements are made. $\boldsymbol{b}_t \in \mathbb{R}^P$ is a snapshot of the measured signal. $\boldsymbol{S} \in \mathbb{R}^{D \times T}$ represents unknown current sources at D candidate locations distributed over the cortex. The orientations of the current densities are restricted to be perpendicular to the cortical mesh. Thus the dimension