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Minimum variance beamformer weights revisited

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ABSTRACT

Adaptive minimum variance beamformers are widely used analysis tools in MEG and EEG. When the target brain activity presents in the form of spatially localized responses, the procedure usually involves two steps. First, positions and orientations of the sources of interest are determined. Second, the filter weights are calculated and source time courses reconstructed. This last step is the object of the current study. Despite different approaches utilized at the source localization stage, basic expressions for the weights have the same form, dictated by the minimum variance condition. These classic expressions involve covariance matrix of the measured field, which includes contributions from both the sources of interest and the noise background. We show analytically that the same weights can alternatively be obtained, if the full field covariance is replaced with that of the noise, provided the beamformer points to the true sources precisely. In practice, however, a certain mismatch is always inevitable. We show that such mismatch results in partial suppression of the true sources if the traditional weights are used. To avoid this effect, the "alternative" weights based on properly estimated noise covariance should be applied at the second, source time course reconstruction step. We demonstrate mathematically and using simulated and real data that in many situations the alternative weights provide significantly better time course reconstruction quality than the traditional ones. In particular, they a) improve source-level SNR and yield more accurately reconstructed waveforms; b) provide more accurate estimates of inter-source correlations; and c) reduce the adverse influence of the source correlations on the performance of single-source beamformers, which are used most often. Importantly, the alternative weights come at no additional computational cost, as the structure of the expressions remains the same.

Introduction

Minimum variance adaptive spatial filters or beamformers (Van Veen et al., 1997; Robinson and Vrba, 1999; Sekihara and Nagarajan, 2008; Greenblatt et al., 2005; Huang et al., 2004; Herdman and Cheyne, 2009) have been widely used for bioelectromagnetic source reconstruction in magnetoencephalography (MEG) and electroencephalography (EEG). Both technologies make it possible to register electromagnetic fields generated by synchronous post-synaptic electric currents in populations of neurons (Hamalainen et al., 1993; Herdman and Cheyne, 2009), using an array of sensors positioned outside the brain. Beamformers allow reconstruction of these neuronal activations based on the MEG/EEG measurements. In principle, such reconstruction can be achieved using a number of methods, each one involving certain assumptions and approximations (see Baillet et al., 2001; Mosher et al., 2003; Greenblatt et al., 2005 for a review). In most cases, it is assumed that electromagnetic field of the brain is produced by a large number of elementary sources, usually approximated by point current dipoles, although other source types may be considered (Limpiti et al., 2006; Jerbi et al., 2002). In the beamformer approach it is further assumed that among all the brain sources there is a relatively small number of those that determine a "signal" part of the measured field. These sources are often referred to as "active", "task-related" sources, "sources of interest", etc. All other brain sources are collectively regarded as background often called "brain noise". This model is justified in many practical situations, especially when studying the task- or stimuli-driven components of the overall brain activity. The goal of the beamformer analysis is to locate the above mentioned sources in the brain, and then reconstruct their time courses by applying a spatial filtering technique. Accordingly, a typical procedure involves two steps. First, locations and orientations of the sources of interest are determined by searching the brain volume (or more precisely, the source parameter space)

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using localizer functions (or simply "localizers"). Second, the filter weights of the beamformer pointing to the found sources are calculated. Both steps are closely related, because incorrect source localization may result in the waveforms reconstructed on the second step become distorted or even meaningless. The first step is much more complicated than the second, and this is where the existing beamforming algorithms differ. For example, localizer functions based on different assumptions have been proposed, and the search procedure itself can be implemented in a number of ways. On the contrary, irrespective of the approach applied at the localization step, *the same* (*or mathematically equivalent*) *expressions for the spatial filter weights are used* once the source parameters were determined (step two). These classic expressions have been known for decades in the radar field (Frost, 1972); works by Robinson and Rose, 1992 and Van Veen et al., 1997 were likely the first ones where this approach was applied to neuroimaging. The expressions follow directly from the "minimum variance" principle and certain linear constraints imposed on the weights. The weights turn out to be fully determined by a) the lead field(s) of the target source(s) and b) the covariance matrix of the field measured by the sensor array when the sources of interest were active. We call it "full" covariance from here on, because it includes contributions from both the sources of interest (the signal) and the noise. The term "noise" here refers to the part of the measured field which is *not* produced by the target sources. It includes environmental, instrumental noise and the brain noise as well. The noise covariance matrix is not needed for the spatial filter construction, but in most cases participates in the localizer functions.

In this paper, we were primarily concerned with the second step of the beamformer approach. We showed analytically that the traditional expressions for the weights can be improved by replacing the full covariance matrix with the noise covariance, and demonstrated this in simulations and using human MEG data. Specifically, both the traditional and new expressions yield identical results provided the source parameters were determined precisely, while the noise-based ones yield more accurate reconstructions in the presence of modeling, source localization and other errors. At the same time, the localizers applied at the first step of the beamformer analysis should still be constructed using the traditional weights based on the full covariance.

Methods

Throughout the paper, vectors and matrices are specified in lower- and uppercase bold letters, respectively (i.e. $\boldsymbol{a}$ and $\boldsymbol{A}$), scalars, including components of vectors and matrices — in regular letters (i.e. t, P, A_{ij}). The symbol "^" denotes an estimate of some quantity, as opposed to its true value. Subscript "0" added to a symbol means that corresponding quantity is calculated when the true source parameters are substituted. For convenience, we used notation adopted in Moiseev et al. (2011), Moiseev and Herdman, (2013), as some of the results are used in this study.

EEG/MEG forward model

Assume that EEG or MEG signals are recorded by an array of M sensors. Let $\boldsymbol{b}(t)$ denote an M-dimensional column vector of sensor readings at time t, and suppose that $\boldsymbol{b}(t)$ is produced by $n < M$ sources of interest $s_i(\boldsymbol{\theta}^i, t)$, $i = 1, \ldots, n$, and noise $\boldsymbol{\nu}(t)$:

$$\boldsymbol{b}(t) = \sum_{i=1}^{n} s_i\left(\boldsymbol{\theta}_o^i, t\right)\boldsymbol{h}^i\left(\boldsymbol{\theta}_0^i\right) + \boldsymbol{\nu}(t). \tag{1}$$

In Eq. (1), $s_i(\boldsymbol{\theta}^i, t)$ is the instantaneous amplitude of the i-th source; vector $\boldsymbol{\theta}^i$ denotes a set of parameters defining the source. This set depends on the source type. Without loss of generality the sources are assumed to be point current dipoles from here on. In this case $\boldsymbol{\theta}^i$ consists of a source position $\boldsymbol{r}^i$ and a unit orientation vector $\boldsymbol{u}^i$: $\boldsymbol{\theta}^i = \{\boldsymbol{r}^i, \boldsymbol{u}^i\}$. M-dimensional column vectors $\boldsymbol{h}^i(\boldsymbol{\theta}^i)$, $\boldsymbol{\nu}(t)$ define the source lead fields (forward solutions) and the noise measured by the array, respectively. Specifically, $\boldsymbol{\nu}$ describes that part of the measured field $\boldsymbol{b}$ which is *not* produced by the sources of interest s_i and includes environmental, instrumental noise as well as *fields generated by other brain sources* (the brain noise). Further on, we use the term "sources" when referring to the sources of interest, understanding that all other brain activations are accounted for in the noise field $\boldsymbol{\nu}$. Requirement $n < M$ (so called "low rank" assumption) is necessary to derive the weights expressions (see Sekihara and Nagarajan, 2008). Both the true source parameters $\boldsymbol{\theta}_0^i$ and the true source forward solution $\boldsymbol{h}_0^i$ do not change with time. $s_i(\boldsymbol{\theta}_0^i, t)$ and $\boldsymbol{\nu}(t)$ are assumed to be uncorrelated zero-mean stationary random processes: $\langle s_i(\boldsymbol{\theta}_0^i, t)\rangle = 0$, $\langle \boldsymbol{\nu}(t)\rangle = 0$, $\langle s_i(\boldsymbol{\theta}_0^i, t)\boldsymbol{\nu}\rangle = 0$, where the angle brackets denote statistical averaging.

Minimum variance spatial filter solution

In the linear spatial filter approach, an estimate $\hat{s}_i$ of the unknown amplitude $s_i(\boldsymbol{\theta}_0^i, t)$ is sought in the form of a weighted sum of the sensor array readings:

$$\hat{s}_i(\boldsymbol{\Theta}, t) = \sum_{m=1}^{M} w_m^i(\boldsymbol{\Theta}) b_m(t) = \boldsymbol{w}^i(\boldsymbol{\Theta})^T \boldsymbol{b}(t) \tag{2}$$

where M-dimensional column vector $\boldsymbol{w}^i$ defines the beamformer weights for the source "i": $\boldsymbol{w}^i = \{w_1^i, \ldots, w_M^i\}^T$; superscript "$T$" denotes transposition. Vectors $\boldsymbol{w}^i$ depend on parameters $\boldsymbol{\Theta} = \{\boldsymbol{\theta}^1, \ldots, \boldsymbol{\theta}^n\}$ of the "target" sources the beamformer points to, but do not change with time. For *adaptive minimum variance beamformers* (Frost, 1972, Van Veen et al., 1997; Robinson and Vrba, 1999; Sekihara et al., 2004; Sekihara and Nagarajan, 2008), the weights $\boldsymbol{w}^i$ are found by minimizing average total power of sources P, reconstructed using Eq. (2). The power is defined as $P = \sum_{i=1}^{n} \langle \hat{s}_i^2 \rangle = \sum_{i=1}^{n} \boldsymbol{w}^{iT} \langle \boldsymbol{b}\boldsymbol{b}^T \rangle \boldsymbol{w}^i = tr(\boldsymbol{W}^T \boldsymbol{R} \boldsymbol{W})$. Here $\boldsymbol{R} = \langle \boldsymbol{b}\boldsymbol{b}^T \rangle$ is $(M \times M)$ covariance matrix of the field, measured by the sensor array, which we also call the *full* covariance. Matrix $\boldsymbol{W}$ has the weight vectors of individual sources $\boldsymbol{w}^i$ as its columns, $\boldsymbol{W} = \{\boldsymbol{w}^1, \ldots, \boldsymbol{w}^n\}$, and has dimensions $(M \times n)$. Minimization of power is performed subject to *unit gain constraints*: $\boldsymbol{w}^{iT}\boldsymbol{h}^i = 1$, $i = 1, \ldots, n$, where $\boldsymbol{h}^i$ denote the forward solutions of the sources the beamformer is steered to. Unit gain constraints ensure that minimization of P does not reduce the signals received from the sources of interest. Additionally, if $n > 1$, the weight vector $\boldsymbol{w}^i$ of source i should be orthogonal to forward solutions of all other targets: $\boldsymbol{w}^{iT}\boldsymbol{h}^j = 0$, $j \neq i$, $i, j = 1, \ldots, n$ (*zero-gain constraints*). Zero-gain constraints prevent sources $j \neq i$ from contributing to the reconstructed time course $\hat{s}_i(\boldsymbol{\Theta}, t)$. Both unit and zero gain constraints are combined by a single matrix equation $\boldsymbol{W}^T\boldsymbol{H} = \boldsymbol{I}_n$, where lead field matrix $\boldsymbol{H}$ has target forward solutions $\boldsymbol{h}^i$ as its columns, $\boldsymbol{H} = \{\boldsymbol{h}^1, \ldots, \boldsymbol{h}^n\}$, and $\boldsymbol{I}_n$ is n-dimensional identity matrix. A well-known classic formula for weights $\boldsymbol{W}$ minimizing the total power P subject to the above constraints is (Frost, 1972; Sekihara and Nagarajan, 2008; Van Veen et al., 1997):

$$\boldsymbol{W} = \boldsymbol{R}^{-1}\boldsymbol{H}\left(\boldsymbol{H}^T\boldsymbol{R}^{-1}\boldsymbol{H}\right)^{-1}. \tag{3}$$

Eq. (3) or its sub-cases is commonly used by all popular minimum variance filters, including single-source scalar and vector beamformers (Robinson and Vrba, 1999; Van Veen et al., 1997; Sekihara and Nagarajan, 2008; Huang et al., 2004), evoked beamformers (Robinson, 2004; Cheyne et al., 2007), as well as multi-source versions of those (Brookes et al., 2007; Dalal et al., 2006; Diwakar et al., 2011; Moiseev et al., 2011, Moiseev and Herdman, 2013). Sometimes, additional normalizations are applied to the weights themselves, or to the lead fields in Eq. (3), or to both (see Sekihara and Nagarajan, 2008 for examples). Such normalizations neither change the structure of the expression