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Non-Negative Spherical Deconvolution (NNSD) for estimation of fiber Orientation Distribution Function in single-/multi-shell diffusion MRI

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ABSTRACT

Spherical Deconvolution (SD) is commonly used for estimating fiber Orientation Distribution Functions (fODFs) from diffusion-weighted signals. Existing SD methods can be classified into two categories: 1) Continuous Representation based SD (CR-SD), where typically Spherical Harmonic (SH) representation is used for convenient analytical solutions, and 2) Discrete Representation based SD (DR-SD), where the signal profile is represented by a discrete set of basis functions uniformly oriented on the unit sphere. A feasible fODF should be non-negative and should integrate to unity throughout the unit sphere $\mathbb{S}^2$. However, to our knowledge, most existing SH-based SD methods enforce non-negativity only on discretized points and not the whole continuum of $\mathbb{S}^2$. Maximum Entropy SD (MESD) and Cartesian Tensor Fiber Orientation Distributions (CT-FOD) are the only SD methods that ensure non-negativity throughout the unit sphere. They are however computational intensive and are susceptible to errors caused by numerical spherical integration. Existing SD methods are also known to overestimate the number of fiber directions, especially in regions with low anisotropy. DR-SD introduces additional error in peak detection owing to the angular discretization of the unit sphere. This paper proposes a SD framework, called Non-Negative SD (NNSD), to overcome all the limitations above. NNSD is significantly less susceptible to the false-positive peaks, uses SH representation for efficient analytical spherical deconvolution, and allows accurate peak detection throughout the whole unit sphere. We further show that NNSD and most existing SD methods can be extended to work on multi-shell data by introducing a three-dimensional fiber response function. We evaluated NNSD in comparison with Constrained SD (CSD), a quadratic programming variant of CSD, MESD, and an *L1*-norm regularized non-negative least-squares DR-SD. Experiments on synthetic and real single-/multi-shell data indicate that NNSD improves estimation performance in terms of mean difference of angles, peak detection consistency, and anisotropy contrast between isotropic and anisotropic regions.

Introduction

Diffusion MRI (dMRI) non-invasively reveals the microstructure of white matter by capturing the diffusion patterns of water molecules. The most widely used dMRI approach, Diffusion Tensor Imaging (DTI), captures one fiber direction per voxel and is incapable of describing complex diffusion processes due to its Gaussian diffusion assumption (Johansen-Berg and Behrens, 2009). In view of this, many High Angular Resolution Diffusion Imaging (HARDI) (Tuch et al., 2002) methods have been developed in recent years to characterize non-Gaussian diffusion and compute quantities such as the Ensemble Average Propagator (EAP) (Wedeen et al., 2005; Descoteaux et al., 2010; Cheng et al., 2010b; Özarslan et al., 2009; Cheng et al., 2012), diffusion Orientation Distribution Function (dODF) (Tuch, 2004; Hess et al., 2006; Descoteaux et al., 2007; Aganj et al., 2010; Cheng et al., 2010a; Cheng et al., 2012), and fiber Orientation Distribution Function (fODF) (Tournier et al., 2004; Tournier et al., 2007; Alexander, 2005; Jian and Vemuri, 2007; Dell'Acqua et al., 2007; Dell'Acqua et al., 2010; Landman et al., 2012; Weldeselassie et al., 2012).

Spherical deconvolution (SD) has been shown to be effective for estimating the fODF by assuming that the measured diffusion-weighted signal can be obtained via spherically convolving a latent fODF with a fiber response function estimated from voxels known to be traversed by a single fascicle (Tournier et al., 2004, 2007; Jian and Vemuri, 2007; Johansen-Berg and Behrens, 2009). The fODF can hence be recovered via an inverse problem by deconvolving the signal with the estimated fiber response function. The local peaks (maxima) of the fODF give the corresponding fiber directions. SD methods can be classified into two

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categories, 1) Continuous Representation based SD (CR-SD), which is normally based on the Spherical Harmonic (SH) basis (Tournier et al., 2004, 2007; Anderson, 2005) and 2) Discrete Representation based SD (DR-SD), which is based on a discrete mixture of rotated versions of the fiber response function (Jian and Vemuri, 2007; Dell'Acqua et al., 2007, 2010; Landman et al., 2012).

Existing SD methods in both continuous and discrete representation categories share some common limitations. First, they often result in false-positive fiber directions (Tournier et al., 2004, 2007; Alexander, 2005; Johansen-Berg and Behrens, 2009; Jian and Vemuri, 2007; Landman et al., 2012; Weldeselassie et al., 2012), especially in low-anisotropy gray matter and cerebrospinal fluid (CSF) regions. Second, they normally fall short in ensuring that the estimated fODF is a proper probability density function, because non-negativity and unit integral throughout the unit sphere are not explicitly enforced. Most SD methods, including the popular Constrained SD (CSD) (Tournier et al., 2007) and all DR-SD methods (Jian and Vemuri, 2007; Dell'Acqua et al., 2007; Dell'Acqua et al., 2010; Landman et al., 2012), consider non-negativity only on discretized points but not the whole continuum of the unit sphere $\mathbb{S}^2$. To the best of our knowledge, Maximum Entropy SD (MESD) (Alexander, 2005) and Cartesian Tensor Fiber Orientation Distributions (CT-FOD) (Weldeselassie et al., 2010; Weldeselassie et al., 2012) are the only existing methods that ensure non-negativity throughout $\mathbb{S}^2$. However, they are computationally inefficient and rely on the error-prone process of numerical spherical integration. Ad-hoc normalization is also employed in these methods to obtain fODFs with unit integral. Some methods estimate continuously non-negative dODFs (Schwab et al., 2012; Cheng et al., 2012; Krajsek and Scharr, 2012) and EAPs (Cheng et al., 2012) using eigenvalue distribution of spherical functions and square root representation. But to our knowledge, none of these methods has been proposed to estimate continuously non-negative fODF in a SD framework. Third, for estimation of the fODF with reasonable accuracy, DR-SD methods (Jian and Vemuri, 2007; Dell'Acqua et al., 2007, 2010; Landman et al., 2012) require a significant amount of rotated fiber response functions along directions that are distributed densely on the unit sphere, significantly increasing the dimensionality and the time cost of the optimization problem. Furthermore, since for DR-SD methods (Jian and Vemuri, 2007; Dell'Acqua et al., 2007, 2010; Landman et al., 2012) the local peaks (maxima) of the fODF are detected from discretized points on the unit sphere, the angular resolution is limited.

To our knowledge, existing SD methods deal only with single-shell data (i.e., single b-value) and do not consider the radial component of diffusion. With advances in dMRI, multi-shell data are increasingly available (e.g., Human Connectome Project (HCP) (Sotiropoulos et al., 2013)). For example, the HCP Q1 data come with three b-values ($b = 1000/2000/3000$ s/mm^2). Recent estimation methods such as the ball-stick model (Jbabdi et al., 2012), Q-Ball Imaging (Aganj et al., 2010), and other multi-shell HARDI methods (Assemlal et al., 2011; Cheng et al., 2010a, 2010b; Descoteaux et al., 2010; Özarslan et al., 2009) demonstrated that ODF, EAP as well as fiber directions can be estimated with greater accuracy from multi-shell data compared with single-shell data. However, there is currently no existing work on how to perform SD on multi-shell data.

In this paper, we propose a method called Non-Negative Spherical Deconvolution (NNSD) to estimate fODF from both single- and multi-shell data.

The main contributions of this paper are summarized as follows:

- NNSD is the first SH-based SD method to guarantee non-negativity throughout $\mathbb{S}^2$, not only on discretized points on the unit sphere, as in CSD (Tournier et al., 2007). In NNSD, non-negativity is achieved by representing the square root of the fODF as a linear combination of SH basis functions. Compared with non-SH methods like MESD and CT-FOD, which also guarantee non-negativity on the whole $\mathbb{S}^2$, NNSD is significantly faster due to the use of closed-form expression for spherical convolution. Compared with CSD which suppresses negative values in discrete samples using iteratively re-weighted regularization, the non-negativity constraint in NNSD is built into its fODF representation and hence NNSD works well even without any regularization.
- In addition to the non-negativity constraint, NNSD reduces spurious peaks by implementing Riemannian gradient descent with an adaptive stopping condition. As a result, the anisotropy values of the fODFs estimated by NNSD in gray and white matter regions exhibit a large contrast. Existing SD methods result in many false-positive peaks and hence high anisotropy in regions that are less anisotropic.
- Compared with traditional single-shell methods, we show that multi-shell data can be used for fODF estimation with greater robustness.
- Performance evaluation using real data is difficult due to the lack of ground truth. We propose in this paper a measure called peak consistency (PC) for quantitative fODF evaluation without exact knowledge of the ground truth.

Part of this work has been reported in our conference paper (Cheng et al., 2013a, 2013b). Herein, we provide additional examples, results, derivations, and insights that are not part of this conference publication.

The rest of the paper is organized as follows. SD theory and algorithms are reviewed in Section Theory. Section Spherical deconvolution revisited provides an overview of existing SD methods using single-/multi-shell data, i.e. CSD (Tournier et al., 2007), a variant of CSD based on quadratic programming, MESD (Alexander, 2005), and DR-SD via *L1* regularized non-negative least-squares fitting (L1-NNLS) (Jian and Vemuri, 2007; Landman et al., 2012). Section Non-Negative Spherical Deconvolution (NNSD) describes NNSD and the associated Riemannian gradient descent algorithm. Two stopping strategies for Riemannian gradient descent are discussed. Section Methods furnishes evaluation details, including fiber response function estimation (Section Estimation of fiber response function), peak detection (Section Peak detection), synthetic data generation (Section Synthetic data simulation and evaluation), and peak consistency evaluation (Section Real data evaluation via peak consistency). In Section Experiments, NNSD is evaluated in comparison with the methods discussed in Section Spherical deconvolution revisited. Section Discussion provides additional discussions on various aspects of NNSD. Section Conclusion concludes this paper.

Theory

Spherical deconvolution revisited

In this section, we describe CSD, MESD, and L1-NNLS, which were originally proposed for single-shell data, and generalize them for multiple-shell data. We also proposed a new implementation of CSD using quadratic programming.

Constrained SD (CSD)

SD (Tournier et al., 2004; Anderson, 2005) methods assume that the measured signal in each voxel is the product of convolving a latent fODF with an axisymmetric fiber response function. For $\mathbf{u} \in \mathbb{S}^2$, the fODF is represented as

$$\Phi(\mathbf{u}) = \sum_{l=0}^{L} \sum_{m=-l}^{l} f_{lm} Y_l^m(\mathbf{u}), \tag{1}$$

and the axisymmetric 3D fiber response function along the z-axis is represented as

$$H(q\mathbf{u}|(0,0,1)) = \sum_{l=0}^{L} h_l(q) Y_l^0(\mathbf{u}), \tag{2}$$