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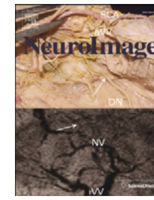

Multi-core beamformers: Derivation, limitations and improvements

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ABSTRACT

Minimum variance beamformers are popular tools used in EEG and MEG for analysis of brain activity. In recent years new multi-source beamformer methods were developed, including the Dual-Core Beamformer (DCBF) and its enhanced version (eDCBF). Both techniques should allow modeling of correlated brain activity under a wide range of conditions. However, the mathematical justification given is based on single-source results and computer simulations, which do not provide an insight into the assumptions involved and the limits of their applicability. Current work addresses this problem. Analytical expressions relating actual source parameters to those obtained with the DCBF and eDCBF are derived, and rigorous conclusions regarding the accuracy of the DCBF/eDCBF reconstructions are made. In particular, it is shown that DCBF accurately identifies source coordinates, but amplitudes and orientations are only correct for high SNRs and fully correlated sources. In contrast, eDCBF source localization is inaccurate, but if the source positions are found precisely, eDCBF allows perfect reconstruction for arbitrary SNRs. If the source positions are approximate, the reconstruction errors are generally larger for higher SNR values. The eDCBF results can be improved by using global unbiased localizer functions and an alternative way of estimating source orientations.

Introduction

Linear adaptive spatial filters are becoming popular source-imaging tools for electroencephalography (EEG) and magnetoencephalography (MEG) studies. These filters, also known as beamformers, reconstruct electrical activity in locations inside the brain based on the signals recorded by an array of sensors positioned outside the head (MEG) or mounted on the head surface (EEG). In particular, minimum variance beamformers (Greenblatt et al., 2005; Herdman and Cheyne, 2009; Huang et al., 2004; Robinson and Vrba, 1999; Sekihara and Nagarajan, 2008; Van Veen et al., 1997) proved to be very effective in a variety of practical situations for estimating neural generators. Mathematically the problem of reconstructing electrical sources inside the brain based on the fields observed outside the head is ill posed and cannot be solved without making additional assumptions about the sources (Baillet et al., 2001; Greenblatt et al., 2005). In particular, commonly used Linearly Constrained Minimum Variance (LCMV) filters (Van Veen et al., 1997), Spatial Aperture Magnetometry (SAM) filters (Robinson and Vrba, 1999) and their variations (Herdman and Cheyne, 2009; Huang et al., 2004; Sekihara and Nagarajan, 2008) are based on the assumption that the sources are uncorrelated or statistically orthogonal.

In practice this assumption is often violated because when a cognitive or perceptual task is performed many brain regions are involved and often exhibit correlated activity. A commonly described example of source correlation occurs as a result of bilateral sensory areas being activated simultaneously for visual or auditory stimuli (see for example Quraan and Cheyne, 2010; Herdman et al., 2003), especially in situations where synchronized activations are sustained for long time periods, as seen in the auditory steady state response (ASSR) measurements (Herdman et al., 2003).

Significant source correlations adversely affect performance of the conventional minimum variance beamformers. In particular, correlated sources tend to cancel each other leading to decreased signal-to-noise ratio (SNR) and distorted time courses (Hillebrand and Barnes, 2005; Sekihara et al., 2002; Van Veen et al., 1997). Technically, this happens because the filter weight coefficients are found independently for each location, implicitly assuming that the source at the target location is the only one. In other words, conventional beamformers are "single-source". To improve the beamformer performance in the presence of correlated activity, other sources should somehow be accounted for in the filter-weight calculations, making the beamformer "multisource".

One way to implement multisource beamformers is to derive the weights using a linear combination of the lead fields of possible sources (Brookes et al., 2007). This approach allowed accurate localization of a pair of highly correlated sources but it becomes computationally expensive with an exponentially increasing amount of calculations, as the number of potential sources grows. Diwakar et al. (2011a) suggested a modification of this method, which treats a pair of the source lead fields as a single higher-dimensional lead field and also reduces the

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number of calculations involved. The authors called this a dual-core beamformer (DCBF).

Another approach is the multiple constrained minimum variance (MCMV) beamformer, originating from the field of radar detection (Frost, 1972) and successfully applied to the EEG/MEG inverse problem in the recent years (Dalal et al., 2006; Hui et al., 2010; Moiseev et al., 2011; Popescu et al., 2008; Quraan and Cheyne, 2010). The MCMV beamformer weights are calculated under the constraint that the filter gains for signals originating from other source locations, or even from extended brain regions are (approximately) zero. This way the correlated interference at the target location, which is the main cause of problems for the single-source beamformers, is eliminated. A newer version of DCBF called the "enhanced" DCBF (eDCBF, Diwakar et al., 2011b) also imposes such constraints on the weights, and therefore belongs to the MCMV family. Although multi-source beamformers proved successful for reconstruction of bilateral activations of visual and auditory sensory areas (Popescu et al., 2008; Quraan and Cheyne, 2010) and in functional connectivity analysis (Hui et al., 2010), they have not become widely accepted practice likely due to at least two reasons.

First, in most cases the source locations are not known *a priori*, and the beamformer itself should be used to find them. For the single-source filters this is done by an exhaustive search over the entire brain volume. In multi-source beamforming such "brute force" approaches are currently unfeasible given the typical computing power in most laboratories, as the number of dimensions of the searched parameter space grows exponentially with the number of sources.

Second, the choice of brain activity measure, or the *localizer function* used to find sources is not obvious. The localizer function should reach its maximum when the true source parameters are matched. For the single-source beamformers such *unbiased* localizers are known, for example, the pseudo-Z ratio and the neural activity index (Greenblatt et al., 2005; Sekihara et al., 2005). In the multi-source cases various forms of the single-source localizers were still applied (Dalal et al., 2006; Popescu et al., 2008; Quraan and Cheyne, 2010) although their unbiased property was not established. Recently several unbiased multi-source localizers were derived by Moiseev et al. (2011).

The present paper investigates the validity of DCBF and eDCBF (Diwakar et al., 2011a,b) methods. Each of them addresses the above problems differently and suggests its own localizer function, source search algorithm, and amplitudes and orientations reconstruction procedure. Although these suggested solutions could improve computational speed, the justification given is based on the single-source not multi-source results, and is mainly supported by numerical experiments. In this work, we investigated DCBF and eDCBF methods analytically. For the general case of arbitrary number of correlated sources, we analyzed the DCBF and eDCBF localizer functions to verify that they are unbiased, and tested the accuracy of amplitudes, orientations, and correlations. However, we did not investigate the DCBF/eDCBF source search algorithms as these are general numerical methods not specific to the bioelectromagnetic inverse problem. Furthermore, we looked at non-ideal situations where the source localization or covariance matrix estimates might be approximate. Analytical results were confirmed using computer simulations.

Methods

Minimum variance linear filter solutions

The following notation is used throughout this paper. Vectors and matrices are specified in lower and upper-case bold letters respectively (i.e. **a** and **A**), while scalar quantities, including the components of vectors and matrices — in regular letters (for example, t, a_i, A_{ij}, P). Additionally, we use the symbol "hat" ("^") to distinguish an estimate of some quantity rather than its true value, which might be not known.

Let $\boldsymbol{b}(t)$ denote the M-dimensional column vector of the EEG or MEG sensor readings at time t, where M is the number of sensors. Assume that $\boldsymbol{b}(t)$ is generated by $n_0<M$ possibly correlated point dipolar sources $s_i(\boldsymbol{\theta}^i,t), i=1,\ldots,n_0$ plus noise $\boldsymbol{\nu}(t)$:

$$\boldsymbol{b}(t) = \sum_{i=1}^{n_0} s_i\left(\boldsymbol{\theta}^i, t\right)\boldsymbol{h}^i\left(\boldsymbol{\theta}^i\right) + \boldsymbol{\nu}(t). \tag{1}$$

In Eq. (1) $s_i(\boldsymbol{\theta}^i,t)$ is an instantaneous amplitude of the source. Vector $\boldsymbol{\theta}^i$ includes both source position $\boldsymbol{r}^i$ and orientation $\boldsymbol{u}^i$: $\boldsymbol{\theta}^i=\{\boldsymbol{r}^i,\boldsymbol{u}^i\}$, $\boldsymbol{u}^i$ being a unit vector. The M-dimensional column vectors $\boldsymbol{h}^i(\boldsymbol{\theta}^i)$, $\boldsymbol{\nu}(t)$ define source lead fields and the noise measured by the array, respectively. Additionally, we assume that the source parameters $\boldsymbol{\theta}^i=\{\boldsymbol{r}^i,\boldsymbol{u}^i\}$ do not change with time and that the zero-mean random processes $s_i(\boldsymbol{\theta}^i,t)$, $\boldsymbol{\nu}(t)$ are stationary and uncorrelated: $\langle s_i(\boldsymbol{\theta}^i,t)\rangle=0$, $\langle \boldsymbol{\nu}(t)\rangle=0$, $\langle s_i\boldsymbol{\nu}\rangle=0$, $i=1,\ldots,n_0$, where the angle brackets denote statistical averaging.

Given the model described by Eq. (1), the goal is to solve the *inverse problem* of finding parameters $\boldsymbol{\theta}^i$ and the amplitudes $s_i(\boldsymbol{\theta}^i,t)$ that explain the measured field $\boldsymbol{b}(t)$. A solution to this problem in the linear adaptive minimum variance filter approach may be summarized as follows (Robinson and Vrba, 1999; Sekihara and Nagarajan, 2008; Sekihara et al., 2004; Van Veen et al., 1997). An estimate $\hat{s}_i$ for the unknown amplitude of a source i is sought in the form of a weighted sum of the sensor array readings:

$$\hat{s}_i(\boldsymbol{\Theta}, t) = \sum_{m=1}^{M} w_m^i(\boldsymbol{\Theta}) b_m(t) \equiv \left(\boldsymbol{w}^i(\boldsymbol{\Theta})\right)^{\boldsymbol{T}}\boldsymbol{b}(t) \tag{2}$$

Here $\boldsymbol{w}^i=\{w_1^i,\ldots,w_M^i\}^T$ is the M-dimensional column vector of the beamformer weights for the source i, and the superscript "T" denotes transposition. The weights $\boldsymbol{w}$ are supposed to be time-independent, but they do depend on a set of other parameters encapsulated in a parameter vector $\boldsymbol{\Theta}$. This set varies depending on the beamformer type, as will be discussed shortly. Irrespective of a concrete form of $\boldsymbol{\Theta}$, the weights of the *minimum variance* beamformers are found by minimizing the total reconstructed source power $P=\sum_{i=1}^{n}\left\langle \hat{s}_i^2\right\rangle$ subject to certain constraints. According to Eq. (2), P can be also written as $P=\sum_{i=1}^{n}\boldsymbol{w}^{iT}\boldsymbol{R}\boldsymbol{w}^i = Tr\left(\boldsymbol{W}^{\boldsymbol{T}}\boldsymbol{R}\boldsymbol{W}\right)$, where $\boldsymbol{R}=\langle \boldsymbol{b}\boldsymbol{b}^T\rangle$ is the $(M\times M)$ covariance matrix of the field measured by the sensor array, and the $(M\times n)$ matrix $\boldsymbol{W}$ has the weight vectors of the individual sources as its columns: $\boldsymbol{W}=\{\boldsymbol{w}^1,\ldots,\boldsymbol{w}^n\}$. Note that the assumed number of sources n (also called the beamformer order) might differ from the true number of sources n_0, which is often unavailable. Specifically, postulating that $n=1$ constitutes the conventional single-source case. In this work we follow the DCBF/eDCBF assumption that the actual number of sources is known: $n=n_0$.

The minimization of power is performed requesting that $\boldsymbol{w}^{iT}\boldsymbol{h}^i=1$, which means that the filter reconstructs the amplitude of the unit source at the target location exactly (the *unit gain constraint*). Nothing else is needed for the single-source case $n=1$. If $n>1$, weights $\boldsymbol{w}^i$ are also required to be orthogonal to the fields of other sources $\boldsymbol{h}^j$, to ensure signal separation: $\boldsymbol{w}^{iT}\boldsymbol{h}^j=0, j\neq i$ (the *zero gain constraints*). The full set of constraints may be written as a single matrix equation $\boldsymbol{W}^T\boldsymbol{H}=\boldsymbol{I}_n$, where $(M\times n)$ matrix $\boldsymbol{H}$ has forward solutions of individual sources as its columns: $\boldsymbol{H}=\{\boldsymbol{h}^1,\ldots,\boldsymbol{h}^n\}$, and $\boldsymbol{I}_n$ is an n-dimensional identity matrix. The weights $\boldsymbol{W}$ that minimize power P, subject to the above constraints, constitute the MCMV beamformer solution and are equal to (Frost, 1972; Sekihara and Nagarajan, 2008; Van Veen et al., 1997):

$$\boldsymbol{W} = \boldsymbol{R}^{-1}\boldsymbol{H}\boldsymbol{S}^{-1} \tag{3}$$