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A note on the phase locking value and its properties

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ABSTRACT

We investigate the properties of the Phase Locking Value (PLV) and the Phase Lag Index (PLI) as metrics for quantifying interactions in bivariate local field potential (LFP), electroencephalography (EEG) and magnetoencephalography (MEG) data. In particular we describe the relationship between nonparametric estimates of PLV and PLI and the parameters of two distributions that can both be used to model phase interactions. The first of these is the von Mises distribution, for which the sample PLV is a maximum likelihood estimator. The second is the relative phase distribution associated with bivariate circularly symmetric complex Gaussian data. We derive an explicit expression for the PLV for this distribution and show that it is a function of the cross-correlation between the two signals. We compare the bias and variance of the sample PLV and the PLV computed from the cross-correlation. We also show that both the von Mises and Gaussian models are suitable for representing relative phase in application to LFP data from a visually-cued motor study in macaque. We then compare results using the two different PLV estimators and conclude that, for this data, the sample PLV provides equivalent information to the cross-correlation of the two complex time series.

Introduction

Information processing in the brain involves coordination of neuronal populations distributed throughout the cerebral cortex (Horwitz, 2003; Tononi and Edelman, 1998). Detecting and quantifying the interactions between these neuronal populations can lead to important insights into the dynamic networks that underlie human brain function. Noninvasive electrophysiological mapping with the electroencephalogram (EEG) and magnetoencephalogram (MEG), as well as invasive recordings in patients and nonhuman primates, provides data that we can use to explore these interactions. Electrophysiological signals can be usefully characterized in terms of their oscillatory components either through band-pass filtering into the standard frequency bands (delta, theta, alpha, beta, and gamma) or using broadband spectral representations of the data. Interactions can then be analyzed using measures of within and between frequency-band coupling between electrode or magnetometer pairs. If EEG or MEG data are first mapped back onto the cortex using an inverse mapping procedure (Baillet et al., 2001), then we can also compute interactions between time series averaged over cortical regions of interests (ROIs).

In this paper we restrict attention to within-band coupling computed between pairs of electrodes, magnetometers or cortical ROIs. The most widely used measure defines interaction in terms of coherence, a complex measure of phase and amplitude similarity computed as a function of frequency (Challis and Kitney, 1991; Klein et al., 2006; Nunez et al., 1997). An alternative class of measures considers only the relative phase through computation of a phase locking value between the two signals (Tass et al., 1998). Phase locking is a fundamental concept in dynamical systems that has been used in control systems (the phase-locked loop) and in the analysis of nonlinear, chaotic and nonstationary systems. Since the brain is a nonlinear dynamical system, phase locking is an appropriate approach to quantifying interaction. A more pragmatic argument for its use in studies of LFPs (local field potentials), EEG and MEG is that it is robust to fluctuations in amplitude that may contain less information about interactions than does the relative phase (Lachaux et al., 1999; Mormann et al., 2000).

The most commonly used phase interaction measure is the Phase Locking Value (PLV), the absolute value of the mean phase difference between the two signals expressed as a complex unit-length vector (Lachaux et al., 1999; Mormann et al., 2000). If the marginal distributions for the two signals are uniform and the signals are independent then the relative phase will also have a uniform distribution and the PLV will be zero. Conversely, if the phases of the two signals are strongly coupled then the PLV will approach unity. For event-related studies we would expect the marginal to be uniform across trials unless the phase is locked to a stimulus. In that case, we may have nonuniform marginals which could in principle lead to false indications of phase locking.

When comparing electrode pairs that share a common reference or overlapping lead field sensitivities, or when investigating cortical

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current density maps of limited resolution, the PLV suffers from sensitivity to linear mixing in which the same source can contribute to both channels. In these cases, the PLV can indicate an apparent phase locking with the relative phases concentrated around zero. Stam et al. (2007) proposed an alternative measure, the Phase Lag Index (PLI) that is robust to the common source problem. PLI quantifies the asymmetry of the relative phase distribution about zero and so will produce large values only when the relative phase is peaked away from zero.

In this paper we first define nonparametric estimates of PLV and PLI, and consider the bias intrinsic in the sample PLV estimator. We derive an expression for the unbiased estimator of the squared PLV and show equivalence to the Pairwise Phase Consistency (PPC) metric recently proposed by Vinck et al. (2010). We then investigate the relationship between PLV and PLI and two possible parametric distributions that can be used to model relative phase. The first of these, the von Mises distributions, is the maximum entropy distribution over the class of circular distributions (Jammalamadaka and Sengupta, 2001). The second model is the relative phase distribution associated with complex circularly symmetric Gaussian processes. This model is appropriate for complex signals generated from jointly Gaussian real signals through use of the Hilbert transform. The relative phase distribution is obtained by marginalizing the joint Gaussian distribution with respect to the amplitude of the two complex signals. We derive closed-form expressions for the relationship between PLV and the parameters of the von Mises and Gaussian models.

Invasive microelectrode recordings can be used to investigate both multiunit activity, which reflects axonal firing rates, and the local field potentials (LFPs) associated with dendritic and volume conduction currents. In this paper we are concerned with the application of PLV and PLI measures to LFPs as well as noninvasive EEG and MEG measurements that similarly result from dendritic and volume conduction currents. We use LFP recordings from a macaque monkey study (Bressler et al., 1999) to investigate whether the von Mises and Gaussian distributions are appropriate for modeling relative phase between pairs of electrodes. We then compare the ability of two different estimators of PLV, associated respectively with the von Mises and Gaussian models, to detect phase locking between electrodes.

The goal of this work is to clarify the relationships between nonparametric estimators of PLV and PLI and two well-known parametric distributions that could be used to model phase interactions. A second goal is to investigate the relationship between PLV and cross-correlation when analyzing LFP data. We begin by stating, and where appropriate deriving, these relationships. We then present computational simulations and analysis of experimental LFP data using different PLV estimators.

Measures of phase synchronization

The phase locking value and phase lag index

Phase synchronization between two narrow-band signals is frequently characterized by the Phase Locking Value (PLV). Consider a pair of real signals $s_1(t)$ and $s_2(t)$, that have been band-pass filtered to a frequency range of interest. Analytic signals $z_i(t) = A_i(t)e^{j\phi_i(t)}$ for $i=\{1,2\}$ and $j=\sqrt{-1}$ are obtained from $s_i(t)$ using the Hilbert transform:

$$z_i(t) = s_i(t) + j\mathrm{HT}(s_i(t)) \tag{1}$$

where $\mathrm{HT}(s_i(t))$ is the Hilbert transform of $s_i(t)$ defined as

$$\mathrm{HT}(s_i(t)) = \frac{1}{\pi}\mathrm{P.V.}\int_{-\infty}^{\infty}\frac{s_i(t)}{t-\tau}d\tau \tag{2}$$

and P.V. denotes Cauchy principal value. Once the analytic signals are defined, the relative phase can be computed as

$$\Delta\phi(t) = \arg\left(\frac{z_1(t)z_2^*(t)}{|z_1(t)||z_2(t)|}\right). \tag{3}$$

The instantaneous PLV is then defined as (Celka, 2007; Lachaux et al., 1999)

$$\mathrm{PLV}(t) \triangleq \left|\mathrm{E}\left[e^{j\Delta\phi(t)}\right]\right| \tag{4}$$

where E[.] denotes the expected value. The PLV takes values on [0, 1] with 0 reflecting the case where there is no phase synchrony and 1 where the relative phase between the two signals is identical in all trials. PLV can therefore be viewed as a measure of trial to trial variability in the relative phases of two signals. In this work we use the Hilbert transform but the continuous Morlet wavelet transform can also be used to compute complex signals, producing separate band-pass signals for each scaling of the wavelet. Quiroga et al. (2002) and Le Van Quyen et al. (2001) have shown that both approaches yield similar results.

When computing synchrony between pairs of electrodes or cortical locations, nonzero PLVs can arise from a single source contributing to both signals as a result of either volume conduction in channel space or limited spatial resolution in the case of cortical current density maps (Amor et al., 2005; David et al., 2002; Guevara et al., 2005; Nunez et al., 1997; Tass et al., 1998; Vinck et al., 2011). In this case of direct linear mixing there is no phase lag between the two signals potentially resulting in a large value of PLV. Linear mixing can therefore easily be mistaken for phase locking between distinct signals. To distinguish these two conditions we need a different measure of phase locking that is zero in the case of linear mixing but nonzero when there is a consistent nonzero phase difference between the two signals. The Phase Lag Index (PLI) (Stam et al., 2007) achieves this goal by quantifying the asymmetry of the distribution of relative phase around zero and is defined as

$$\mathrm{PLI} \triangleq |E[sign(\Delta\phi(t))]|. \tag{5}$$

PLI takes values on the interval [0, 1] and is zero if the distribution of relative phase is symmetric about 0 or π.

In practice PLV and PLI are typically estimated by averaging over trials and/or time (Aviyente et al., 2010; Lachaux et al., 1999, 2000; Mormann et al., 2000; Stam et al., 2007). For notational convenience, we will drop the explicit dependence on t in the following. A nonparametric estimate of PLV can be computed by approximating Eq. (4) by averaging over trials:

$$\hat{\mathrm{PLV}}_{\mathrm{sample}} \triangleq \left|\frac{1}{N}\sum_{n=1}^{N}e^{j\Delta\phi_n(t)}\right| \tag{6}$$

where n indexes the trial number and N is the total number of trials. The estimator generalizes in an obvious way to incorporate averaging over multiple time samples. The corresponding nonparametric estimator for PLI is

$$\hat{\mathrm{PLI}}_{\mathrm{sample}} \triangleq \left|\frac{1}{N}\sum_{n=1}^{N}sign(\Delta\phi_n(t))\right|. \tag{7}$$

In the following section we consider the relationship between PLV and PLI and the parameters of two alternative probability distributions that can be used to characterize phase interactions: the von Mises and the bivariate circularly symmetric Gaussian.