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Research in Perspective

How do symbolic and non-symbolic numerical magnitude processing skills relate to individual differences in children's mathematical skills? A review of evidence from brain and behavior

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ABSTRACT

Many studies tested the association between numerical magnitude processing and mathematics achievement, but results differ depending on the number format used. For symbolic numbers (digits), data are consistent and robust across studies and populations: weak performance correlates with low math achievement and dyscalculia. For non-symbolic formats (dots), many conflicting findings have been reported. These inconsistencies might be explained by methodological issues. Alternatively, it might be that the processes measured by non-symbolic tasks are not critical for school-relevant mathematics. A few neuroimaging studies revealed that brain activation during number comparison correlates with children's mathematics achievement level, but the consistency of such relationships for symbolic and non-symbolic processing is unclear. These neurocognitive data provided ground for educational interventions, which seem to have positive effects on children's numerical development in (a)typical populations.

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1. Introduction

One important way in which cognitive neuroscience has made successful connections to educational research is by drawing attention to the importance of numerical magnitude processing as a foundation for higher-level numerical and mathematical skills (e.g., [10,19]). Over the last decade, this has fueled research aimed at investigating the relationship between individual differences in numerical magnitude processing skills and arithmetic achievement in typically developing children as well as studies probing whether children with atypical mathematical development or developmental dyscalculia (DD) are impaired in their abilities to process numerical magnitudes. Such research is beginning to lay the foundations for the design and evaluation of educational interventions that foster numerical magnitude processing.

One of the outstanding questions in this emerging body of research is whether processing magnitudes in either symbolic

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(digits) or non-symbolic (dots) formats or both is crucial for successful mathematics achievement. Such research can pinpoint more precisely the mathematical content that should be included in specific interventions.

Beyond educational applications, establishing whether symbolic or non-symbolic numerical magnitude processing skills, or both, are predictive of children's mathematics achievement is of theoretical importance too. While non-symbolic representations of numerical magnitudes are thought to be shared across species and can already be measured in early infancy [13], symbolic representations are uniquely human and relatively recent cultural inventions to provide abstract representations of numerical magnitude. Thus, by investigating the relationship between, on the one hand symbolic and non-symbolic numerical magnitude processing and, on the other, children's mathematical achievement, larger questions concerning the role of evolutionary ancient skills for the acquisition of uniquely human number skills and representations can also be constrained. In this contribution, we provide an integrative review of the existing body of data that has dealt with this question.

Box 1–: Magnitude comparison tasks

The nature of numerical representations is typically explored by using magnitude comparison tasks. In a standard **nonsymbolic** magnitude comparison task, participants are shown two dot arrays – or sequences of sounds – and asked to select the more numerous. The difficulty of making this decision is manipulated by varying the ratio or the numerical distance between the two arrays. For example, it is more difficult to distinguish 12 and 9 dots (ratio 0.75; numerical distance 3) than it is to distinguish 12 and 6 dots (ratio 0.5; numerical distance 6).

Typical measures of performance include overall accuracy, response time (RT), ratio or distance effects or the *w* index. This Weber fraction (*w*) can be calculated on the basis of the participants' performance across different ratios, and provides a measure of the acuity of ANS representations. Individuals with a smaller *w* have more precise ANS representations than those with a larger *w*.

To increase the possibility that participants use the number of dots rather than visual characteristics of the displays (e.g., dot size, density, total area), the dot arrays are typically constructed in such a manner so that these characteristics do not correlate with numerosity across the task, i.e. dot size, density and area vary across the experiment. However, recent data by Gebuis and Reynvoet [28] indicate that it is impossible to perfectly control for these non-numerical parameters and that the number of items in a set cannot be extracted independently of visual cues. While the existing studies all controlled for non-numerical parameters in their experimental design, the degree to which some visual properties of the stimuli are controlled for varies between them and this might also account for the differences in the results obtained. In other words, it is unclear how participants use the various non-numerical visual characteristics of the stimuli to guide their decision as to which array of dots is larger and how this process might differ between children who have various levels of mathematical competence. On the other hand, the data by Gebuis and Reynvoet [28] also call into question the degree to which non-symbolic number processing can truly be measured.

Symbolic comparison tasks typically have the same format, except that the quantities are represented as Arabic digits, or in some studies, number words. Similar effects of distance or ratio on performance are observed when people perform this task.

2. Development of non-symbolic number processing

The nature and role of typically developing children's magnitude representations have been commonly explored with magnitude comparison tasks (Box 1). Nonsymbolic (dot) comparison tasks are frequently thought to index the precision or acuity of representations within the approximate number system (ANS), a system which allows individuals to represent and process numerical magnitude information. Representations within the ANS are noisy and become increasingly imprecise with increasing magnitude. Individuals with more precise ANS representations perform more accurately and faster on magnitude comparison tasks and they show smaller effects of ratio or distance. Typically developing children also show an increase in the precision of ANS representations over developmental time (e.g., [30]).

It has been hypothesized that performance on non-symbolic magnitude comparison tasks is related to mathematics achievement, but the evidence to support this proposal is mixed (Table 1). A number of studies have found that dot comparison performance is related to prior, concurrent and future mathematics achievement. However, many studies have failed to find such a significant relationship (see Table 1 for a summary). One possible explanation for these contrasting findings is that there is no standardized version of the dot comparison task. Studies vary in the size of the dot arrays, the way in which visual characteristics of the dots are controlled, the length of time the displays are presented and the performance measures used. This final point is particularly important as the range of possible measures includes mean accuracy, (median) RT, Weber Fraction (*w*) estimates, and distance or ratio effects, which may be calculated in a number of ways on the basis of accuracy or RT. These measures capture different aspects of participants' performance, they are not interchangeable and may show different relationships with mathematics achievement [53,62]. However, as shown in Table 1, studies that have or have not found a significant relationship cannot be easily differentiated by factors such as the dot comparison measure employed or the range of numbers used in a

Table 1
The nature of the relationship between nonsymbolic (dot) comparison task performance and mathematics achievement in typically developing participants. The dot comparison measure(s) used and number range of the task are given in brackets.

Relationship between dot comparison performance and mathematics	
Significant	Nonsignificant
Children	Children
Halberda et al. [32] [*w*; 5–16][a]	Holloway et al. [33] [NDE; 1–9]
Mundy et al. [53] [acc; 1–9]	Mundy et al. [53] [NDE; 1–9]
Inglis et al. [34] [*w*; 5–22]	Soltesz et al. [73] [acc, RT, NRE; 4–20]
Libertus et al. [42] [acc, *w*, RT; 4–15]	Lonnemann et al. [45] [NDE; 4–6]
Mazzocco et al. [50] [acc, *w*; 1–14][a]	Ferreira et al. [26] [acc; 20–44]
Bonny et al. [5] [*w*, acc; 4–12]	Sasanguie et al. [68] [RT/error, NDE; 1–9]
Libertus et al. [43] [acc, *w*, RT; 4–15]	Sasanguie et al. [69] [RT/error, NDE; 1–9][a]
	Vanbinst et al. [75] [NDE; 1–9]
	Fuhs et al. [27] [acc; 1–30]
	Kolkman et al. [37] [acc; 1–100]
	Sasanguie et al. [67] [*w*, acc; 6–26][a]
Adults	Adults
Lyons et al. [47] [*w*; 1–9]	Inglis et al. [34] [*w*; 9–70]
Halberda et al. [31] [*w*, RT; 5–20]	Castronovo et al. [16] [*w*; 12–40]
Libertus et al. [44] [*w*; 5–20]	Price et al. [62] [w, NDE; 6–40]
Lourenco et al. [46] [acc; 5–14]	

Acc=accuracy; NDE=numerical distance effect; NRE=numerical ratio effect; RT=response time; *w*=estimates of Weber fraction.

[a] Longitudinal data.