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# Neurodevelopment of relational reasoning: Implications for mathematical pedagogy

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ABSTRACT

Reasoning ability supports the development of mathematics proficiency, as demonstrated by correlational and longitudinal evidence, and yet this skill is not emphasized in traditional elementary mathematics curricula. We propose that targeting reasoning skills from elementary school onward could help more students succeed in advanced mathematics courses. Here, we review the links between reasoning and mathematics, discuss the neural basis and development of reasoning ability, and identify promising school curricula.



## 1. Introduction

Mathematics achievement in school acts as a gatekeeper for academic and career success [1], preventing students who fail courses such as algebra from entering careers in science, technology, and many areas of business. This issue is cause for concern at a global scale [2], and so it is vital that we understand and address the factors that determine why some students succeed in mathematics while others fail. Educational research has identified several key factors, from choice of curriculum and teacher quality [3,4,5] to home environment and cultural dynamics [6,7,8,9].

We argue here that an additional factor that influences proficiency in mathematics is a student's capacity for *relational reasoning*, or the ability to jointly consider multiple sets of relations between mental representations. Relational reasoning is essential to algebra [10] and helpful in learning many elementary mathematical concepts [11,12]. In this paper we review the theoretical and psychometric links between relational reasoning and mathematics, and present neurodevelopmental evidence for the importance of emphasizing relational reasoning in elementary mathematics instruction.

## 2. Relational reasoning and its role in mathematics

Relational reasoning is a fundamental aspect of what psychologists traditionally call fluid reasoning, or the ability to solve problems in novel situations [13]. The study of relational reasoning distinguishes between first-order and second-order (or higher-order) relationships. A first-order comparison describes the relation between two individual mental representations, whereas a second-order comparison integrates two (or more) sets of first-order relations. A propositional analogy is a good example: in determining whether chain is to link as bouquet is to flower, one must first identify the relationships between each pair, and then compare the nature of those relationships to each other. Semantic and spatial relationships can be structured similarly to create tasks that elicit the same essential relational reasoning skill (Fig. 1 A and B).

Cognitive scientists have long studied relational reasoning in these domain-general contexts, under the assumption that domain-general skills carry over to domain-specific contexts. We hypothesize that the capacity for relational reasoning is a critical foundation for learning mathematical concepts. To illustrate the role of relational reasoning in mathematics, we take the example of algebra. A key difference between advanced and average algebra learners is whether they view the equal sign (=) relationally or operationally [10]. A relational definition of the equal sign emphasizes the *equivalent relationship* between the expressions on either side of the equal sign (Fig. 1 C). An operational definition involves only the computational aspect. For example, when completing a calculation

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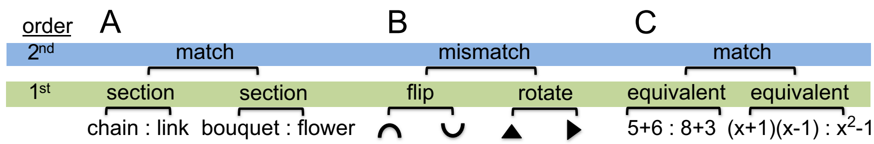


**Fig. 1.** Three examples of relational reasoning tasks, with first-order relationships highlighted in green and second-order in blue. (A) Verbal analogy in which each first-order relationship describes one section of a larger entity. Because these relationships match across the two pairs, the analogy is valid. (B) The shapes in these two pairs do not have the same first-order relationship. (C) Expressions of mathematical equivalence. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

indicated by an expression on the left, the equal sign announces the answer on the right, as in $5\times 10+27-35=42$.

The problem with an operational understanding of the equal sign is that it is insufficient for solving complex algebraic functions, which may have more than one solution, and must be solved by manipulating both sides of the equation. These algebraic calculations are meaningful only if the student holds a relational view of the equal sign [14]. However, traditional elementary math curricula rarely present the equal sign in a relational context [15], and thus many students struggle when introduced to the concept in algebra [16].

## 3. Evidence of correlation between reasoning and mathematics

Understanding of the equal sign is but one illustration of the centrality of relational reasoning in mathematics. There is strong evidence for a more general correlation between these skills. Several studies involving broad batteries of cognitive ability found relational reasoning to be strongly correlated with mathematics performance, above effects of other cognitive factors [17], and across various age ranges [18,19]. These data are strengthened by recent longitudinal analyses that indicate a developmental link between reasoning skills and math achievement. For example, Primi, Ferrão and Almeida [20] found that 11–14-year-olds who had higher relational reasoning scores than their peers at the outset of the study showed greater annual rates of improvement in an independent mathematics assessment. Relational reasoning skill has similarly been shown to be a significant predictor of mathematical skill nine months later in 6-year-olds [21] and 18 months later in 6–18-year-olds [22].

## 4. A relational account of the link between reasoning and mathematics

The data reviewed above provide strong empirical support for a link between relational reasoning and mathematics performance. According to Cattell's investment hypothesis (1987), the link is due to relational reasoning, a component of ‘fluid intelligence,’ providing a scaffold on which to build all domain-specific skills. Yet we posit a more concrete explanation for the correlation between relational reasoning and mathematics. White, Alexander, and Daugherty [23] point out that from an information-processing perspective, analogical and mathematical reasoning require the same elemental cognitive functions, which could explain the correlations between reasoning and mathematics performance observed at a given point in time. However, as noted above, longitudinal data go beyond this conclusion by showing that current reasoning ability is a good predictor of mathematics performance several years later, even after accounting for the strong relationship between reasoning ability measured at the two time-points [22,20].

We theorize that the emerging ability to reason relationally forms the foundation for mathematical conceptual development

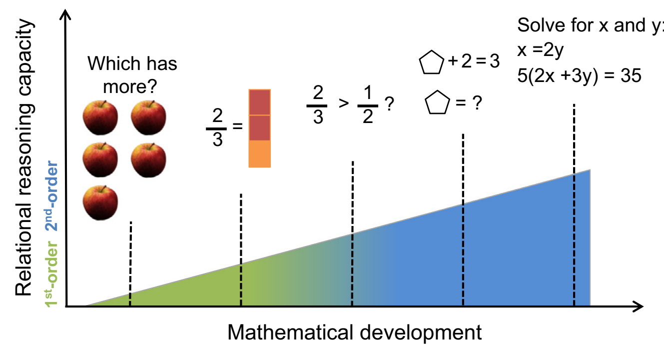


**Fig. 2.** Theoretical model depicting how the development of relational reasoning supports the acquisition of mathematical knowledge. Learning the meaning of number words is a process of comparing a number to the next one in the sequence. Understanding fractions requires the representation of 1st-order relations between numerators and denominators. Comparing fractions requires evaluation of a 2nd-order relation. Pre-algebra tasks require representation of complex relations between known values, unknown values, and operations. Algebra problems include variables and complex systems of equations that must be solved in relation to each other.

throughout the school years (Fig. 2). To learn the meaning of number words, young children must grasp the differences in magnitude and order that the number words imply [24]. They do so through a process of learning to distinguish “one” from “more than one”, and iteratively adding “two” and “three” to their repertoire before grasping the mapping between number word order and increasing magnitudes [25]. Thus, learning the meaning of number words requires first-order comparison of each number and the next one in the sequence. Four to six years later, when students encounter fractions, this comparison becomes even more explicit; fractions are defined and notated by a first-order relationship between the numerator and the denominator. Comparing two fractions requires evaluation of a second-order relationship by determining how the relationship between one numerator and its denominator differs from that between another numerator–denominator pair.

The next major milestone is pre-algebra, such as the task to solve for an unknown number. Even simple equations such as the one shown in Fig. 2 depict somewhat complex relationships between the known and unknown numbers, and suggest the use of an operation (subtraction) that is inversely related to the one displayed (addition). These relationships become higher-level in algebra, with complex expressions and systems of equations required to find the value of two unknown numbers. To master algebra, a student must grasp the concept of a variable, which *represents any number* that satisfies specific relational arguments. Therefore, over the course of mathematical development, children progress from defining numbers as first-order relationships, to making second-order value comparisons, to resolving complex systems of first- and second-order relations involving known, unknown and variable quantities and inverse operations.

Thus, we hypothesize that improvements in relational reasoning over childhood and adolescence support students' ability to reason about increasingly complex mathematical relations: