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Research in Perspective

Skills underlying mathematics: The role of executive function in the development of mathematics proficiency

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ABSTRACT

The successful learning and performance of mathematics relies on a range of individual, social and educational factors. Recent research suggests that executive function skills, which include monitoring and manipulating information in mind (working memory), suppressing distracting information and unwanted responses (inhibition) and flexible thinking (shifting), play a critical role in the development of mathematics proficiency. This paper reviews the literature to assess concurrent relationships between mathematics and executive function skills, the role of executive function skills in the performance of mathematical calculations, and how executive function skills support the acquisition of new mathematics knowledge. In doing so, we highlight key theoretical issues within the field and identify future avenues for research.

Contents

1. Introduction

Children's underachievement in mathematics is a consistent and significant problem [33] with 21% of 11-year-olds leaving primary school without reaching the mathematics level expected of them, and 5% failing even to achieve the numeracy skills expected of a 7-year-old [45]. These problems endure into adulthood, and it is estimated that a fifth of adults have numeracy skills below the basic level needed for everyday situations [95]. Mathematics ability is crucial for success in Western societies [3] and poor mathematics skills have a bigger impact on life chances than poor literacy [75]. Given the significant economical and societal impact of these problems it is important to understand in detail the processes involved in learning and performing mathematics.

Many factors contribute to differences in mathematics achievement, including attitudes [66], motivation [87], language ability [31] and IQ [67], in addition to social [18], and educational factors [72,74]. It is clear that domain-specific numerical skills and knowledge are important for success with mathematics [43,57], but other cognitive factors also play an important role. In particular, the domain-general skill of holding and manipulating information in mind (working memory) has been found to be critical [76]. Inhibition, the ability to suppress distracting information and unwanted responses [16,40,58,84], and shifting, the ability to flexibly switch attention between different tasks [98], have also been implicated in mathematics achievement. These processes fall under the umbrella of executive function (EF); skills required to monitor and control thought and action (Text box 1). Within the research literature, four types of study have aimed to describe and understand the role that EF skills

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Text box 1–What is executive function?

Executive function (EF) is the name given to the group of processes that allow us to respond flexibly to our environment and engage in deliberate, goal-directed, thought and action. Executive function forms the basis of abilities such as problem solving and flexible thinking and is most likely to be used in the absence of external guidance or when a situation is novel. The study of executive function originated from observations of adults with damage to the frontal lobe of the brain and the study of these patients has led to a very strong link in research between executive processes and frontal lobe function. Executive function skills begin to emerge in infancy [30] but are among the last cognitive abilities to mature, continuing to develop into late adolescence [25,53,65].

The three EF skills most commonly studied, particularly within the developmental literature, are *inhibition*: suppressing distracting information and unwanted responses, *shifting*: flexibly switching between different tasks, and *updating* or *working memory*: monitoring and manipulating information in mind. The majority of studies addressing the role of working memory in mathematics are based on the working memory model of Baddeley and Hitch [8,9]. This comprises an attentional control system (the central executive), supported by two subsidiary slave systems for the short-term storage of verbal and visuospatial information (the phonological loop and visuospatial sketchpad, respectively).

play in supporting mathematics achievement: Correlational studies examining the relationship between mathematics and EF at a single time point; experimental studies exploring the role of EF in performing mathematical calculations, learning or training studies that aim to pinpoint how EF skills support the acquisition of new mathematics knowledge and neuroimaging studies, revealing the neural mechanisms by which EF supports mathematics (Text box 2). Here we review this literature with the aims of synthesising current knowledge and identifying questions for future research.

2. Correlational studies

The majority of studies investigating the role of EF skills in mathematics have used a cross-sectional correlational design. Using a range of measures these studies have demonstrated that working memory accounts for unique variance in written and verbal calculation, as well as mathematical word problems, across a range of different age groups [1,4,5,11,16,20,47,58,60,61,68,71,82,83,90,88,92,99]. Importantly it is the ability to manipulate and update, rather than simply maintain, information in working memory that seems to be critical for mathematics proficiency. This variance cannot be explained by other factors such as age, IQ, mathematics ability, processing speed, reading and language skills ([4,11]; but see [36]).

Further evidence that working memory is important for mathematics comes from children who demonstrate a specific difficulty with mathematics. Two recent meta-analyses have suggested that children with mathematics disabilities have particular difficulty with the central executive component of working memory [26,91], especially when numerical information is involved [6,26]. This highlights an issue with many correlational studies where EF tasks involving numerical stimuli are used, e.g. digit span, a verbal working memory task where a string of numbers have to be recalled forwards or backwards. These measures may overestimate the role of EF skills in mathematics compared to non-numerical tasks because of their domain-specificity [76]. However, the fact that working memory predicts mathematics performance even when non-numeric stimuli (e.g. letters or words) are used demonstrates that this is not the sole determinant of the relationship.

Text box 2–: *EF skills and mathematics in the developing brain*

Neuroimaging methods, in particular functional Magnetic Resonance Imaging (fMRI), have opened up the possibility for researchers to determine how individuals of different ages approach numerical processing. In studies which require children and adults to choose the larger of two digits or sets of dots, or verify simple sums, adults typically show more activity than children in posterior parietal areas of the brain, while children show greater activity in frontal areas such as the medial and inferior frontal gyri [7,21,78]. These developmental changes have been interpreted as increased functional specialisation in the parietal brain areas that support numerical cognition alongside decreased dependence on working memory and attention.

Two more recent studies have investigated changes within childhood, either between 7 and 9 years of age [80], or between children of the same age classified as ‘counters’ or more sophisticated ‘retrievers’ when solving sums [23]. In contrast to previous work, both of these studies found increased activation in some frontal areas for the more advanced group. An increase in ventrolateral prefrontal cortex activity in retrievers versus counters was attributed to cognitive control over retrieval (e.g. the selection of retrieval strategies and inhibition of procedural strategies). Developmental changes from 7 to 9 years were reflected by an increase in activity in left dorsolateral prefrontal cortex, which the authors hypothesised may reflect more precise manipulation of information in working memory in the older children.

The differences between these two sets of studies may be indicative of an increase, followed by a decrease in the recruitment of frontal brain areas in numerical processing with age. Alternatively, it may be attributable to differences in the task being performed: Arithmetic [23,80] may require increased recruitment of frontal brain areas with age or strategy change, whereas frontal areas may be less involved in processing of numeric magnitude with age [7,21,78]. Nevertheless, it remains unclear exactly what the frontal activity across these studies represents, in particular whether it reflects the involvement of EF skills. To demonstrate this conclusively it would be necessary to show that the frontal activity seen in mathematical tasks correlates with activity in the same area on a measure of executive processing.

Fewer studies have investigated the role of inhibition and shifting in mathematics performance and the findings are mixed. The majority of studies suggest that inhibitory control abilities do predict performance in mathematics ([14,16,34,40,58,61,84,92]; but see [5]). A recent meta-analysis demonstrated that shifting ability does predict performance in mathematics [98], however it remains unclear whether shifting is an independent predictor of mathematics over and above general intelligence.

Correlational studies provide convincing evidence of a relationship between EF skills and mathematics, which may be stronger than the relationship between EF skills and other areas of academic performance (Text box 3). The majority of studies have used standardised tests of mathematics which confound factual (e.g. 6+4=10), conceptual (e.g. knowing that addition is the inverse of subtraction) and procedural (e.g. ′carrying′ when adding above 10) knowledge. Yet it is well established that there are complex relationships among these components [10,44]. Furthermore, individuals differ in their profile of performance across these components, and may have strengths in one component but not others [32], suggesting that different mathematics components may rely on differential sets of EF skills. Indeed, there is emerging evidence that the contribution of executive skills may differ across these components. Hecht et al. [47] showed that while working