# Combined electroosmotically and pressure driven flow in soft nanofluidics
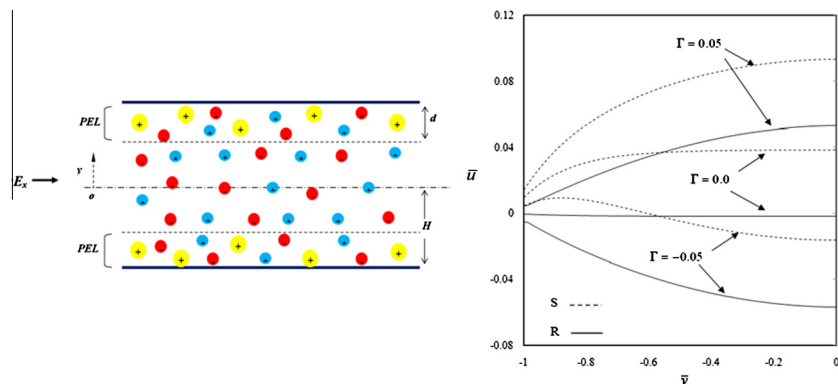
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## GRAPHICAL ABSTRACT



## ARTICLE INFO

## ABSTRACT

The present study is devoted to the analysis of mixed electroosmotic and pressure driven flows through a soft charged nanochannel considering boundary slip and constant charge density on the walls of the slit channel. The sources of the fluid flow are the pressure gradient along the channel axis and the electrokinetic effects that trigger an electroosmotic flow under the influence of a uniformly applied electric field. The polyelectrolyte layer (PEL) is denoted as a fixed charge layer (FCL) and the electrolyte ions can be present both inside and outside the PEL i.e., the PEL–electrolyte interface acts as a semi-penetrable membrane. The Poisson–Boltzmann equation is solved assuming the Debye–Hückel linearization for the low electric potential to provide us with analytical closed form solutions for the conservation equations. The conservation equations are solved to obtain the electric potential and velocity distributions in terms of governing dimensionless parameters. The results for the dimensionless electric potential, the dimensionless velocity and Poiseuille number are presented graphically and discussed in detail.

## 1. Introduction

The electrokinetic transport implies the electrostatic interactions between charged particles in colloidal systems, ionized solutions and interfaces which bring about molecular forces resulting in electric potential in the space of particles. So, the potential of colloidal particles is the influential characteristic in their interfacial electric phenomena, such as the motion of colloidal particles in an electric field. The theory of electrokinetics in systems with micro and nano scales is of utmost importance in analyzing many chemical, biological and medical devices. As a charged surface is brought

into contact with an electrolyte solution, the electrolyte counter ions have the tendency to move towards the charged surface to constitute a layer with a high concentration of counterions. This mobile counterion layer and an immobile surface charge layer form an electric diffuse double layer or EDL near the charge surface. With the application of an external electric field along the surface, a force will be exerted on the ions within the mobile diffuse electric layer. Thus, the ions in the mobile part of the EDL undergo a net migration in the opposite direction and carry the liquid along with them, thereby causing the movement of the liquid [1]. Due to its variety of practical applications, the electrokinetic transport in nanofluidic channels has received a remarkable attention lately [2–8]. Nanofluidic channels with walls made or covered by polyelectrolyte materials are identified as soft nanochannels [9–13]. The polyelectrolyte layer (PEL) is assumed to contain a fixed charge density of ions and then the PEL can also be denoted as a Fixed Charge Layer or an FCL and the electrolyte ions can be present both inside and outside the PEL. Therefore the PEL–electrolyte interface acts as a semi-penetrable membrane. Soft nanochannels have been implemented in energy conversion systems, transistors [14–16] and diodes [17,18], cell membranes [19], nanofluidic and nanoionic valves, quantification of the characteristics of gels and elastomers [20].

To determine the physical mechanism governing the electrokinetic transport of ionized particles and electrolyte solutions in contact with soft surfaces and soft channels, several theoretical attempts have been made. Donath and Voigt [21] pioneered the analysis of electrokinetic theory for a soft surface to investigate the streaming potential. They introduced a model to find the distribution of the fixed charges inside the surface layer without linearization of the Poisson–Boltzmann equation. Ohshima and Kondo [22] developed the previous works by studying the electroosmotic flow in a slit channel with ion-permeable surfaces. Their work was followed by Keh and Liu [23] who analytically investigated the electroosmotic flow in a long capillary with adsorbed polyelectrolyte coverage on the inside wall. In their next work Keh and Ding [24] examined the electroosmotic flow in both circular tubes and capillary slits. In this work the ionogenic reactions between electrolyte solution and the polyelectrolyte porous layer was taken into consideration. In a similar study, Wu and Keh [25] included diffusioosmosis phenomenon in the electroosmotic flow of an electrolyte through a polymer coated slit capillary. Recently Chanda et al. [26] studied electroviscous effects in a pressure driven flow through a soft nanochannel. A comparison between soft and rigid nanochannels was made in order to assess the streaming potential and electroviscous effects of a certain electrolyte in these two kinds of nanochannels. They demonstrated that the streaming potential and electroviscous effects in soft nanochannels are varied remarkably in comparison to rigid nanochannels. In the majority of the studies presented in the literature review, the Poisson–Boltzmann equation is solved assuming Debye–Hückel approximation in order to provide us with analytical closed form solutions for the conservation equations. This assumption is acceptable within the context of electric potential in EDL taking small values. However, Chen and Das [27] enjoyed the advantage of numerical approaches to solve the Poisson–Boltzmann and Navier–Stokes equations beyond Debye–Hückel simplification to ensure that the results are independent of the magnitude of the electric potential.

In case of liquid transport through nanochannels, the Knudsen number can be used to check the continuum assumption which is defined as the ratio of the molecular mean free path ($\lambda$) and the system characteristic length ($L$), i.e. $Kn = \lambda/L$.

In the literature it is repeatedly stated that for $Kn < 0.01$ the continuum model for the flow is correct and when the Knudsen number ranges between 0.01 and 0.1 the slip boundary condition

has to be considered to use continuum model for the flow [28,29]. Since nanofluidic channels might deal with Knudsen numbers smaller than 0.01 the slip boundary condition is taken into account here.

In the open literature predominantly conventional channels are studied and nanofludic channels have to be more focused due to their practical applications. For nanochannels due to large Knudsen number the boundary slip on the walls of the channel should be taken into account which has not been addressed before. Furthermore, a comprehensive comparison between the soft and rigid nanochannels, which makes enough contribution to better understanding of the practical distinctions of these two types, is something that is missing amongst the previous works.

The present study is devoted to the analysis of mixed electroosmotic and pressure driven flows through a soft charged nanochannel considering boundary slip and constant charge density on the walls of the slit channel. The polyelectrolyte layer (PEL) is denoted as a fixed charge layer (FCL) so that the number density of the FCL-ions is assumed to be constant. The Debye–Hückel linearization is considered in obtaining electric potential distributions and the parameters for which this assumption is justifiable are checked. The results for electric potential and velocity are represented in closed-form expressions. The present linear problem can be decomposed into two sub-problems: (i) only an electric field is applied, and (ii) only a pressure gradient field is applied. These two sub-problems have been dealt with extensively in the literature (e.g., [23–25]).

## 2. Mathematical model

Mixed pressure–electrosmotically driven incompressible laminar flow of an electrolyte liquid through an infinitely long soft nanofluidic channel with height of $2H$ is studied. The walls of the channel are subject to constant charge density. This soft nanochannel consists of wall-grafted ion-penetrable charged polyelectrolyte layer (PEL) of thickness $d$. It is indicated in experimental works [30–32] that the PEL thickness ($d$) ranges from 7.8 nm to 3.38 μm depending upon the channel height and the type of the surface material. The ionization of the molecules of the polyelectrolyte layer is the reason of producing the PEL ions which remain entrapped within the PEL. Therefore, these PEL ions are present only within the PEL, whereas the electrolyte ions can be present both inside and outside the PEL. It should be noted that the use of charge neutrality condition at the wall surface–PEL interface is especially valid for a metal surface or an ideally polarizable solid [33]. A schematic of the problem along with the coordinate system is depicted in Fig. 1. We take $y$-axis perpendicular to the walls with its origin on the slit axis at $y = 0$. Also it is assumed that the liquid contains an ideal solution of fully dissociated symmetric salt, and the temperature variation over the channel cross section is negligible compared with the absolute temperature. Using this assumption, the potential field and the charge density may be calculated on the basis of an average temperature.
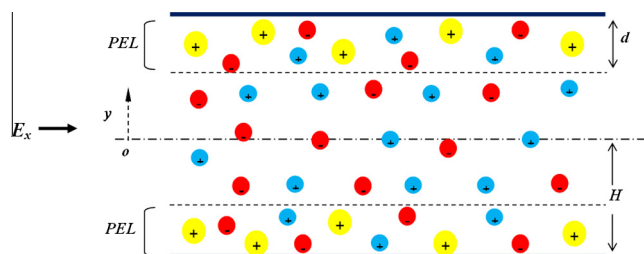


**Fig. 1.** Schematic of the soft nanochannel.