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Analytical and numerical study of the electro-osmotic annular flow of viscoelastic fluids

L.L. Ferrás [a,*], A.M. Afonso [b], M.A. Alves [b], J.M. Nóbrega [a], F.T. Pinho [c]

[a] Institute for Polymers and Composites/I3N, University of Minho, Campus de Azurém, 4800-058 Guimarães, Portugal
[b] Departamento de Engenharia Química, CEFT, Faculdade de Engenharia da Universidade do Porto, Rua Dr. Roberto Frias s/n, 4200-465 Porto, Portugal
[c] Centro de Estudos de Fenómenos de Transporte, Faculdade de Engenharia da Universidade do Porto, Rua Dr. Roberto Frias s/n, 4200-465 Porto, Portugal

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ABSTRACT

In this work we present semi-analytical solutions for the electro-osmotic annular flow of viscoelastic fluids modeled by the Linear and Exponential PTT models. The viscoelastic fluid flows in the axial direction between two concentric cylinders under the combined influences of electrokinetic and pressure forcings. The analysis invokes the Debye-Hückel approximation and includes the limit case of pure electro-osmotic flow. The solution is valid for both no slip and slip velocity at the walls and the chosen slip boundary condition is the linear Navier slip velocity model. The combined effects of fluid rheology, electro-osmotic and pressure gradient forcings on the fluid velocity distribution are also discussed.

1. Introduction

With the evolution of diagnostic tools that capture flow characteristics at the microscale there has been growing evidence of wall slip in experiments using both Newtonian and non-Newtonian fluids [1]. Simultaneously, molecular dynamics has also helped question the veracity of the no-slip law and, nowadays, the wall slip velocity phenomenon in some fluid flows is accepted [2], especially for viscoelastic fluids. In particular, the works of Denn [1] and Lauga et al. [2] provide insights on what has already been done regarding slip velocity measurements and theoretical approaches for Newtonian and non-Newtonian fluids. Kazatchkov and Hatzikiriakos [3] and Hatzikiriakos [4] provide novel physical models that are able to capture the slippery characteristics of certain viscoelastic fluids. All these works concern pressure-driven flows.

For electro-osmotic driven flows the existence of wall slip has been more readily accepted. When an electrolyte solution flows in channels made from dielectric materials, a thin electric double layer (EDL) is spontaneously formed in the vicinity of the wall, where the imbalance of positive and negative ions can be used by an applied electric potential to induce flow along the channel. This layer is usually very small in such a way that the bulk flow can be modeled accurately considering the linear Navier [5] slip boundary condition at the wall [6–11].

In order to ascertain whether the proposed slip models are reliable, analytical solutions and numerical simulations are important tools. Additionally, the analytical solutions can be of major importance in the verification of numerical codes. These two facts, together with the urge of understanding the electro-osmotic flow and the slip phenomenon typical in viscoelastic fluid flows, are the main motives for this work. On what concerns analytical solutions for viscoelastic fluids with slip boundary conditions, we can distinguish two cases: pressure-driven viscoelastic fluid motion; viscoelastic fluid motion driven by a combination of electro-osmotic and pressure forcings.

For Newtonian fluids, Ngoma and Erchiqui [12] investigated numerically the effects of heat flux and boundary slip on electrokinetic flows. Soong et al. [11] analyzed pressure-driven electrokinetic flows in hydrophobic microchannels with emphasis on the slip effects under coupling of interfacial electric and fluid slippage phenomena while Jamaati et al. [13] studied the pressure-driven electrokinetic slip-flow in planar microchannels. For non-Newtonian fluids only analytical solutions under no-slip boundary conditions could be found. Zhao and Yang [14] reported a theoretical analysis of electro-osmotic mobility of non-Newtonian fluids and Afonso et al. [15,16] presented an analytical solution for the mixed electro-osmotic/pressure driven flow of viscoelastic fluids in microchannels and for the case of electro-osmotic flow under symmetric and asymmetric zeta potential, respectively. All these analytical solutions were derived for simple channel flows.

For an annular geometry the literature is rich in analytical solutions for the pressure driven case [17–22] with applications to the oil and gas industries. For the electro-osmotic flow through an annulus, the applications to real life are becoming important in

* Corresponding author.
E-mail addresses: luis.ferras@dep.uminho.pt (L.L. Ferrás), aafonso@fe.up.pt (A.M. Afonso), mmmalves@fe.up.pt (M.A. Alves), mnobrega@dep.uminho.pt (J.M. Nóbrega), fpinho@fe.up.pt (F.T. Pinho).

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biological systems as in electrophoretic separation of proteins and for blending chemical and biological fluids [26]. Regarding analytical studies of such flows we could only find the works of Tsao and Kang et al. [23,24] where the electroosmotic flow of a Newtonian fluid through an annulus was studied for high and low zeta potentials, the work of Goswami and Chakraborty [25] where the authors present semi-analytical solutions for electroosmotic flows of Newtonian fluids with interfacial slip in microchannels of complex cross-sectional shape, including an annular geometry, Jian et al. [26] who analyzed the behavior of time periodic electroosmosis in a cylindrical microannulus, and more recently, Shamshiri et al. [27], who studied electroviscous and thermal effects on the electro-osmotic flow of power-law fluids through an annulus. Analytical solutions for the viscoelastic annular flow case under the influence of both electro-osmotic and pressure driven forcings could not be found in the literature. Given this limitation, in this work we present a semi-analytical solution for the pure axial flow of the Linear and Exponential PTT models [28,29] that is valid for both no-slip and slip boundary conditions.

Although this flow is known to be of interest for industry, we could not find any experimental data. Therefore, we derived a general solution that can cope with various degrees of slip and different classes of fluids. Also, the solution can be easily adapted to other viscoelastic models.

The remaining of this paper presents the relevant set of governing equations, followed by their solutions. A discussion of the effects of the various relevant dimensionless parameters upon the flow characteristics closes this work.

2. Governing equations

The flow of interest is governed by the continuity equation,

$$\nabla \cdot \mathbf{u} = 0 \tag{1}$$

and by the general Cauchy momentum equation,

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho \nabla \cdot \mathbf{u}\mathbf{u} = -\nabla p + \nabla \cdot \boldsymbol{\tau} + \rho_e \mathbf{E} \tag{2}$$

where $\mathbf{u}$ is the velocity vector, p is the pressure, ρ is the density and $\rho_e\mathbf{E}$ represents the electrical force per unit volume acting upon the ions in fluid. This force depends on $\mathbf{E}$, the applied external electric field, and on ρ_e, the net electric charge density. This charge density distribution is a consequence of the distribution of the spontaneously formed electric double layers, which are assumed here not to be affected by the imposed electric field. The deviatoric stress tensor, $\boldsymbol{\tau}$, describes the fluid rheological behavior here given by the simplified Phan-Thien-Tanner (sPTT) model [28,29],

$$f(\mathrm{tr}\boldsymbol{\tau})\boldsymbol{\tau} + \lambda\left(\frac{\partial \boldsymbol{\tau}}{\partial t} + \mathbf{u}\cdot\nabla\boldsymbol{\tau} - [(\nabla\mathbf{u})^T\cdot\boldsymbol{\tau} + \boldsymbol{\tau}\cdot\nabla\mathbf{u}]\right) = \eta(\nabla\mathbf{u} + (\nabla\mathbf{u})^T) \tag{3}$$

where η is the polymer viscosity coefficient, λ is the relaxation time and $f(\mathrm{tr}\boldsymbol{\tau})$ is a function depending on the trace of the stress tensor specifying the various versions of this class of models [28,29],

$$f(\mathrm{tr}\boldsymbol{\tau}) = \begin{cases} 1 + \frac{\varepsilon\lambda}{\eta}\tau_{kk} & \text{linear} \\ \exp\left(\frac{\varepsilon\lambda}{\eta}\tau_{kk}\right) & \text{exponential} \end{cases} \tag{4}$$

As for the boundary conditions, the no-slip boundary condition at the wall is expressed as $\mathbf{u} = \mathbf{0}$, whereas the linear Navier slip law [5], is given by

$$u_{slip} = \pm\mathcal{L}\tau_{rz} \tag{5}$$

where $\mathcal{L}$ is the slip coefficient and τ_{rz} is the wall shear stress. For the inner cylinder wall the plus sign is considered, while for the outer cylinder wall the minus sign is used.

3. Semi-analytical solution

We assume the flow between the two concentric cylinders is fully developed, with the streamwise velocity component in the z direction (the direction of the axes of the cylinders) only depending on the radial coordinate, r. As shown in Fig. 1 the outer cylinder has a radius R, and the radius of the inner cylinder is given by αR with $0 < \alpha < 1$. The gap between the two cylinders is $\delta = R(1-\alpha)$. We further assume that there is no rotation, that the flow is axisymmetric and it is fully developed. For such conditions, continuity, momentum and the constitutive equations can be further simplified.

The axial momentum equation in cylindrical coordinates is given by,

$$\frac{1}{r}\frac{\mathrm{d}(r\tau_{rz})}{\mathrm{d}r} = -\rho_e E_z + p_{,z} \tag{6}$$

where $p_{,z}$ is the constant pressure gradient in the z direction, τ_{rz} is the non-zero shear stress and $E_z \equiv -\mathrm{d}\Phi/\mathrm{d}z$ with $\Phi = \psi + \phi$, where ϕ is the applied streamwise potential and ψ is the equilibrium/induced potential across the cylinders' gap, associated with the interaction between the ions of the fluid and the dielectric wall. The charge density, ρ_e, is related to the electric potential by $\rho_e = -\epsilon\kappa^2\psi(r)$ assuming the Debye-Hückel approximation (at room temperature this limits the potential at the wall to values much smaller than 26 mV), and the induced electric field is given by the solution of the following differential equation [23],

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}\psi}{\mathrm{d}r}\right) = \kappa^2\psi \tag{7}$$

where κ^2 is the Debye-Hückel parameter. For the boundary conditions $\psi(\alpha R) = \zeta_i$ and $\psi(R) = \zeta_o$, the solution of Eq. (7) is given by [23],

$$\psi(r) = \frac{I_0(\kappa r)[\zeta_o K_0(\alpha\kappa R) - \zeta_i K_0(\kappa R)] + K_0(\kappa r)[\zeta_i I_0(\kappa R) - \zeta_o I_0(\alpha\kappa R)]}{I_0(\kappa R)K_0(\alpha\kappa R) - I_0(\alpha\kappa R)K_0(\kappa R)} \tag{8}$$

where $I_0(.)$ and $K_0(.)$ are the modified Bessel functions of first and second kind, respectively.

The induced electric potential is now easily computed by $\rho_e = -\epsilon\kappa^2\psi(r)$ with $\psi(r)$ given by Eq. (8), where ϵ is the dielectric constant of the fluid.

Integration of Eq. (6) results in the following expression for the shear stress,

$$\tau_{rz} = \frac{rp_{,z}}{2} + \epsilon\kappa E_z\frac{K_1(\kappa r)[-\zeta_i I_0(\kappa R) + \zeta_o I_0(\kappa\alpha R)]}{I_0(\kappa R)K_0(\alpha\kappa R) - I_0(\alpha\kappa R)K_0(\kappa R)} + \frac{\epsilon\kappa^2 E_z r}{2} \times \frac{{}_0F_1\left[2, \left(\frac{\kappa r}{2}\right)^2\right][-\zeta_i K_0(\kappa R) + \zeta_o K_0(\kappa\alpha R)]}{I_0(\kappa R)K_0(\alpha\kappa R) - I_0(\alpha\kappa R)K_0(\kappa R)} + \frac{c_1}{r} \tag{9}$$

where ${}_0F_1\left(2, \left(\frac{\kappa r}{2}\right)^2\right)$ is the confluent hypergeometric limit function, which is defined as [30,31]

$${}_0F_1(2,\ z) = \sum_{k=0}^{\infty}\frac{z^k}{k!(k+1)!} \tag{10}$$

and c_1 is the constant of integration that can be determined assuming that $\tau_{rz} = 0$ for $r = \beta R$ with $\alpha < \beta < 1$,

$$c_1 = -\frac{(\beta R)^2 p_{,z}}{2} - \epsilon\kappa E_z\beta R\frac{K_1(\kappa\beta R)(-\zeta_i I_0(\kappa R) + \zeta_o I_0(\kappa\alpha R))}{I_0(\kappa R)K_0(\alpha\kappa R) - I_0(\alpha\kappa R)K_0(\kappa R)} - \frac{\epsilon\kappa^2 E_z(\beta R)^2}{2}\frac{{}_0F_1\left[2, \left(\frac{\kappa\beta R}{2}\right)^2\right](-\zeta_i K_0(\kappa R) + \zeta_o K_0(\kappa\alpha R))}{I_0(\kappa R)K_0(\alpha\kappa R) - I_0(\alpha\kappa R)K_0(\kappa R)} \tag{11}$$

This implies that β has to be determined as part of the solution.

For the pure axial annular flow the shear stress component is given by,

$$f(\tau_{zz})\tau_{rz} = \eta\frac{\mathrm{d}u}{\mathrm{d}r} \tag{12}$$