Journal of Colloid and Interface Science 344 (2010) 513–520

Contents lists available at ScienceDirect

Journal of Colloid and Interface Science

www.elsevier.com/locate/jcis

Steady viscoelastic fluid flow between parallel plates under electro-osmotic forces: Phan-Thien–Tanner model

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ARTICLE INFO

Article history:
Received 18 November 2009
Accepted 8 January 2010
Available online 25 January 2010

Keywords:
Electro-osmotic flow
Parallel plates
PTT model
Viscoelastic fluid
Analytical study
Constitutive flow instability

ABSTRACT

The electro-osmotic flow of a viscoelastic fluid between parallel plates is investigated analytically. The rheology of the fluid is described by the Phan-Thien–Tanner model. This model uses the Gordon–Schowalter convected derivative, which leads to a non-zero second normal stress difference in pure shear flow. A nonlinear Poisson–Boltzmann equation governing the electrical double-layer field and a body force generated by the applied electrical potential field are included in the analysis. Results are presented for the velocity and stress component profiles in the microchannel for different parametric values that characterize this flow. Equations for the critical shear rates and maximum electrical potential that can be applied to maintain a steady fully developed flow are derived and discussed.

1. Introduction

The theoretical analysis of electro-osmotic flows (EOF) of Newtonian fluids in microchannels has been the subject of several studies. Burgreen and Nakache [1] studied the effect of the surface potential on liquid transport through ultrafine capillary slits assuming the validity of the Debye–Hückel linear approximation to the electrical potential distribution under an imposed electrical field. Rice and Whitehead [2] discussed the same problem in a circular capillary. Dutta and Beskok [3] obtained analytical solutions for the velocity distribution, mass flow rate, pressure gradient, wall shear stress, and vorticity in mixed electro-osmotic/pressure driven flows for a two-dimensional straight channel geometry, for small, yet finite symmetric electrical double layers (EDL), relevant for applications where the distance between the two walls of a microfluidic device is about 1–3 orders of magnitude larger than the EDL thickness. Arulanandam and Li [4] and Wang et al. [5] presented a two-dimensional analytical model for the electro-osmotic flow in a rectangular microchannel. Wang et al. [6] derived a semi-analytical solution to study the flow behaviour of periodical electro-osmosis in a rectangular microchannel based on the Poisson–Boltzmann and the Navier–Stokes equations. Zade et al. [7] presented analytical solutions for the heat transfer characteristics of Newtonian fluids under combined pressure and electro-osmotic flow forcing in planar microchannels. Analytical solutions for the two-dimensional, time-dependent as well as time-independent EOF driven by a uniform electric field with non-uniform zeta potential distributions along the walls of a conduit were presented by Qian and Bau [8]. Several other articles can be found in the literature on theoretical studies of EOF with Newtonian fluids in microchannels such as those of Petsev and Lopez [9], Qian and Bau [10], among others.

Biofluids are often solutions of long chain molecules which impart a non-Newtonian rheological behaviour characterized by variable viscosity, memory effects, normal stress effect, yield stress and hysteresis of fluid properties. These fluids are encountered in chips used for chemical and biological analysis or in micro-rheometers.

The theoretical study of electro-osmotic flows of non-Newtonian fluids is recent and has been mostly limited to simple inelastic fluid models, such as the power-law, due to the inherent analytical difficulties introduced by more complex constitutive equations. Examples are the recent works of Das and Chakraborty [11] and Chakraborty [12], who presented explicit relationships for velocity, temperature and concentration distributions in electro-osmotic microchannel flows of non-Newtonian bio-fluids described by the power-law model. Other purely viscous models were analytically investigated by Olivares et al. [13], who considered the existence of a small wall layer depleted of additives and behaving as a Newtonian fluid (the skimming layer), under the combined action of pressure and electrical fields, thus restricting the non-Newtonian behaviour to the electrically neutral region outside the electrical double layer. Very recently these studies were extended to viscoelastic fluids by Afonso et al. [14], who presented analytical

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0021-9797/$ - see front matter
doi:10.1016/j.jcis.2010.01.025

Nomenclature

De_κ	Deborah number, $\lambda\kappa u_{sh}$
e	elementary charge [1.6022×10^{-19} C]
E_x	x-component of imposed electric gradient [V m^{-1}]
$f(\tau_{kk})$	PTT stress coefficient function
H	half-height of the microchannel [m]
k_B	Boltzmann constant [1.3807×10^{-23} J K^{-1}]
L	microchannel length [m]
n_0	ionic number concentration [m^{-3}]
t	time [s]
T	absolute temperature [K]
u	x-component of velocity [m s^{-1}]
u_{sh}	Helmholtz–Smoluchowski velocity [m s^{-1}]
x	axial direction [m]
y	transverse co-ordinate [m]
W	microchannel width [m]
z	valence of ions
Tensors and vectors	
$\mathbf{D}$	rate of deformation tensor [s^{-1}]
$\mathbf{E}$	external applied electric field [V m^{-1}]
$\mathbf{u}$	velocity vector [m s^{-1}]
$\boldsymbol{\tau}$	polymeric extra-stress tensor [Pa]
Greek	
ε	Extensibility parameter of PTT model
$\in$	dielectric constant of the fluid [C V^{-1} m^{-1}]
ϕ	electric potential in the streamwise direction (imposed) [V]
$\dot{\gamma}$	velocity gradient [s^{-1}]
η	polymer viscosity coefficient [Pa s]
κ^2	Debye Hückel parameter [m^{-2}]
λ	relaxation time [s]
λ_D	Debye layer thickness [m]
μ	viscometric viscosity [Pa s]
ρ_e	electric charge density [C m^{-3}]
τ_{xx}, τ_{yy}	normal stresses [Pa]
τ_{xy}	shear stress [Pa]
τ_{kk}	trace of the extra stress tensor [Pa]
ξ	PTT model parameter that accounts for the slip between molecular network and the continuum medium
ψ	potential field in the transverse direction (induced) [V]
ψ_0	wall zeta potential [V]
Subscripts	
c	refers to critical value
κ	refers to Debye–Hückel parameter
sh	refers to Helmholtz–Smoluchowski
s	refers to solvent
Superscript	
$\diamond$	Gordon–Schowalter convected derivative
-	dimensionless quantity

solutions for channel and pipe flows of viscoelastic fluids under the mixed influence of electro-kinetic and pressure forces, using two constitutive models: the Phan-Thien–Tanner model (PTT [15]), with linear kernel for the stress coefficient function and zero second normal stress difference [16], and the kinetic theory based Finitely Extensible Non-linear Elastic model with a Peterlin closure for the average dumbbell spring force (cf. [17]) denoted as FENE-P model. Their analysis [14] was restricted to cases with small electric double-layers, where the distance between the walls of a microfluidic device is at least one order of magnitude larger than the EDL, and the fluid is uniformly distributed across the channel.

Afonso et al. [14] also showed that when the viscoelastic flow is induced by a combination of both electric and pressure potentials, in addition to the contributions from these two isolated mechanisms there is an extra term in the velocity profile that simultaneously combines both forcings, which is absent for the Newtonian fluids where the superposition principle applies. This extra term can contribute significantly to the total flow rate, and appears only when the rheological constitutive equation is non-linear. Afonso et al. [18] extended their earlier study [14] to the flow of viscoelastic fluids under asymmetric zeta potential forcing, whereas Sousa et al. [19] considered the effect of a Newtonian skimming layer for the PTT fluid.

Flow instabilities can occur for a variety of reasons. For instance, they are associated with perturbations to non-linear terms of the governing equations which grow without control. Generally speaking, in electro-osmotic flows in microchannels, flow instabilities can be promoted by oscillating electric fields, as was justified by Boy and Storey [20] among others. They can also be promoted by gradients of conductivity as shown in the experimental study of Lin et al. [21] who also analyzed the problem theoretically and numerically.

In addition to inertial non-linearities, which require high Reynolds number flows, non-Newtonian fluids are also prone to flow instabilities due to non-linearities in their rheological behaviour. For instance, for viscoelastic fluids constitutive instabilities in Poiseuille and Couette flows were observed when the constitutive equation exhibits a non-monotonic behaviour for the shear stress, as reported by Alves et al. [22] for the full PTT model, and by Español et al. [23] and Georgiou and Vlassopoulos [24] for the Johnson–Segalman (JS) constitutive equation [25]. To the best knowledge of the authors this constitutive instability in microchannels under EOF has not yet been studied. There are other viscoelastic flow instabilities not associated with non-monotonic fluid properties, but these are not considered here.

In this study, we extend the work of Afonso et al. [14] considering the full Gordon–Schowalter convective derivative in the PTT model to analyze the steady fully developed flow in the microchannel. We derive expressions for the critical shear rates and Deborah number beyond which constitutive flow instability occurs. The rest of the paper is organised as follows. The physical description of the problem is given in Section 2 while the equations governing the flow are presented in Section 3. The analytical solutions are derived in Section 4. Section 5 discusses the results of the study and the conclusions are presented in Section 6.

2. Physical description of the problem

The geometry under consideration is shown schematically in Fig. 1, where a microchannel is formed between two parallel plates separated by a distance (height) $2H$. The length of the channel is L and the width is W, both assumed to be much larger than the height, i.e., $L, W \gg 2H$. The bottom plate is located at $y = -H$ while the top plate is located at $y = H$. A potential is applied along the axis of the channel which provides the necessary driving force for the flow through electro-osmosis. Due to symmetry of the geometry and flow conditions with respect to the channel mid-