Gently Modulating Optomechanical Systems

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We introduce a framework of optomechanical systems that are driven with a mildly amplitude-modulated light field, but that are not subject to classical feedback or squeezed input light. We find that in such a system one can achieve large degrees of squeezing of a mechanical micromirror—signifying quantum properties of optomechanical systems—without the need of any feedback and control, and within parameters reasonable in experimental settings. Entanglement dynamics is shown of states following classical quasiperiodic orbits in their first moments. We discuss the complex time dependence of the modes of a cavity-light field and a mechanical mode in phase space. Such settings give rise to certifiable quantum properties within experimental conditions feasible with present technology.

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Periodically driven quantum systems exhibit a rich behavior and display nonequilibrium properties that are absent in their static counterparts. By appropriately exploiting time-periodic driving, strongly correlated Bose-Hubbard-type models can be dynamically driven to quantum phase transitions [1], systems can be dynamically decoupled from their environments to avoid decoherence in quantum information science [2], and quite intriguing dynamics of Rydberg atoms strongly driven by microwaves [3] can arise. It has also been muted that such time-dependent settings may give rise to entanglement dynamics in oscillating molecules [4]. A framework of such periodically driven systems is provided by the theory of linear differential equations with periodic coefficients or inhomogeneities, including Floquet's theorem [5].

In this Letter, we aim at transferring such ideas to describe a new and in fact quite simple regime of optomechanical systems, of micromirrors as part of a Fabry-Perot cavity [6–9], and also to one of the settings [10–14] that are the most promising candidates in the race of exploring certifiable quantum effects involving macroscopic mechanical modes. This is an instance of a regime of driving with mildly amplitude-modulated light. We find that in this regime, high degrees of squeezing below the vacuum noise level can be reached, signifying genuine quantum dynamics. More specifically, in contrast to earlier descriptions of optomechanical systems with a periodic time dependence in some aspect of the description, we will not rely on classical feedback based on processing of measurement-outcomes—a promising idea in its own right in a continuous-measurement perspective [15,16]—or resort to driving with squeezed light. Instead, we will consider the plain setting of a time-periodic amplitude modulation of an input light. The picture developed here, based in the theory of differential equations, gives rise to a framework of describing such situations. We find that large degrees of squeezing can be reached (complementing other very recent nonperiodic approaches based on cavity-assisted squeezing using an additional squeezed light beam [17]). It is the practical appeal of this work that such quantum signatures can be reached without the necessity of any feedback, no driving with additional fields, and no squeezed light input (the scheme by far outperforms direct driving with a single squeezed light mode): in a nutshell, one has to simply gently shake the system in time with the right frequency to have the mechanical and optical modes rotate appropriately around each other, reminiscent of parametric amplification, and to so directly certify quantum features of such a system.

Time-dependent picture of system.—Before we discuss the actual time dependence of the driven system, setting the stage, we start our description with the familiar Hamiltonian of a system of a Fabry-Perot cavity of length L and finesse F being formed on one end by a moving micromirror,

$$H = \hbar\omega_c a^\dagger a + \frac{1}{2}\hbar\omega_m(p^2 + q^2) - \hbar G_0 a^\dagger a q + i\hbar \sum_{n=-\infty}^{\infty} (E_n e^{-i(\omega_0+n\Omega)t} a^\dagger - E_n^* e^{i(\omega_0+n\Omega)t} a). \quad (1)$$

Here, ω_m is the frequency of the mechanical mode with quadratures q and p satisfying the usual commutation relations of canonical coordinates, while the bosonic operators a and $a^\dagger$ are associated to the cavity mode with frequency ω_c and decay rate $\kappa = \pi c/(2FL)$. $G_0 = \sqrt{\hbar/(m\omega_m)}\omega_c/L$ is the coupling coefficient of the radiation pressure, where m is the effective mass of the mode of the mirror being used. Importantly, we allow for any periodically modulated driving at this point, which can be expressed in such a Fourier series, where $\Omega = 2\pi/\tau$ and $\tau > 0$ is the modulation period. The main frequency of the driving field is ω_0 while the modulation coefficients $\{E_n\}$ are related to the power of the associated sidebands $\{P_n\}$ by $|E_n|^2 = 2\kappa P_n/(\hbar\omega_0)$. The resulting dynamics under this Hamiltonian together with an unavoidable coupling of the mechanical mode to a thermal reservoir and cavity losses gives rise to the quantum Langevin equation in the reference frame rotating with frequency ω_0, $\dot{q} = \omega_m p$, and

$$\begin{aligned} \dot{p} &= -\omega_m q - \gamma_m p + G_0 a^\dagger a + \xi, \\ \dot{a} &= -(\kappa + i\Delta_0)a + iG_0 aq \\ &\quad + \sum_{n=-\infty}^{\infty} E_n e^{-in\Omega t} + \sqrt{2\kappa}a^{\text{in}}, \end{aligned} \tag{2}$$

where $\Delta_0 = \omega_c - \omega_0$ is the cavity detuning. γ_m is here an effective damping rate related to the oscillator quality factor Q by $\gamma_m = \omega_m/Q$. The mechanical (ξ) and the optical (a^{in}) noise operators have zero mean values and are characterized by their auto correlation functions which, in the Markovian approximation, are $\langle \xi(t)\xi(t') + \xi(t')\xi(t)\rangle/2 = \gamma_m(2\bar{n}+1)\delta(t-t')$ and $\langle a^{\text{in}}(t)a^{\text{in}\dagger}(t')\rangle = \delta(t-t')$, where $\bar{n} = [\exp(\frac{\hbar\omega_m}{k_B T}) - 1]^{-1}$ is the mean thermal phonon number. Here, we have implicitly assumed that such an effective damping model holds [18], which is a reasonable assumption in a wide range of parameters including the current experimental regime.

Semiclassical phase space orbits.—Our strategy of a solution will be as follows: we will first investigate the classical phase space orbits of the first moments of quadratures. We then consider the quantum fluctuations around the asymptotic quasiperiodic orbits, by implementing the usual linearization of the Heisenberg equations of motion [11,12] (excluding the very weak driving regime). Exploiting results from the theory of linear differential equations with periodic coefficients, we can then proceed to describe the dynamics of fluctuations and find an analytical solution for the second moments.

If we average the Langevin equations (2), assuming $\langle a^\dagger a\rangle \simeq |\langle a\rangle|^2$, $\langle aq\rangle \simeq \langle a\rangle\langle q\rangle$ (true in the semiclassical driving regime we are interested in), we have a nonlinear differential equation for the first moments. Far away from instabilities and multistabilities, a power series ansatz in the coupling G_0 $\langle O\rangle(t) = \sum_{j=0}^{\infty} O_j(t)G_0^j$ is justified, where $O = a,\ p,\ q$. If we substitute this expression into the averaged Langevin equation (2), we get a set of recursive differential equation for the variables $O_j(.)$. The only two nonlinear terms in Eq. (2) are both proportional to G_0, therefore, for each j, the differential equation for the set of unknown variables $O_j(.)$ is a *linear* inhomogeneous system with constant coefficients and τ-periodic driving. Then, after an exponentially decaying initial transient (of the order of $1/\gamma_m$), the asymptotic solutions O_j will have the same periodicity of the modulation [5], justifying the Fourier expansion

$$\langle O\rangle(t) = \sum_{j=0}^{\infty}\sum_{n=-\infty}^{\infty} O_{n,j} e^{in\Omega t} G_0^j. \tag{3}$$

Substituting this in Eq. (2), we find the following recursive formulas for the time independent coefficients $O_{n,j}$, $q_{n,0} = p_{n,0} = 0$, $a_{n,0} = E_{-n}/(\kappa + i(\Delta_0 + n\Omega))$, corresponding to the zero coupling $G_0 = 0$, while for $j \geq 1$, we have

$$\begin{aligned} q_{n,j} &= \omega_m \sum_{k=0}^{j-1}\sum_{m=-\infty}^{\infty} \frac{a^*_{m,k} a_{n+m,j-k-1}}{\omega_m^2 - n\Omega^2 + i\gamma_m n\Omega}, \\ p_{n,j} &= \frac{in\Omega}{\omega_m} q_{n,j}, \quad a_{n,j} = i\sum_{k=0}^{j-1}\sum_{m=-\infty}^{\infty} \frac{a_{m,k} q_{n-m,j-k-1}}{\kappa + i(\Delta_0 + n\Omega)}, \end{aligned} \tag{4}$$

Within the typical parameter space, considering only the first terms in the double expansion (3), corresponding to the first sidebands, leads to a good analytical approximation of the classical periodic orbits, see Fig. 1. On physical grounds, this is expected to be a good approximation, since $G_0 \ll \omega_m$, and because high sidebands fall outside the cavity bandwidth, $n\Omega > 2\kappa$. What is more, the decay behavior of E_n related to the smoothness of the drive inherits a good approximation in terms of few sidebands.

Quantum fluctuations around the classical orbits.—We will now turn to the actual quantum dynamics taking first moments into account separately when writing any operator as $O(t) = \langle O\rangle(t) + \delta O(t)$. The frame will hence be provided by the motion of the first moments. In this reference frame, as long as $|\langle a\rangle| \gg 1$, the usual linearization approximation to (2) can be implemented. In what follows, we will also use the quadratures $\delta x = (\delta a + \delta a^\dagger)/\sqrt{2}$ and $\delta y = -i(\delta a - \delta a^\dagger)/\sqrt{2}$, and the analogous input noise quadratures x^{in} and y^{in}. For the vector of all quadratures we will write $u = (\delta q, \delta p, \delta x, \delta y)^T$, with $n = (0, \xi, \sqrt{2\kappa}x^{\text{in}}, \sqrt{2\kappa}y^{\text{in}})^T$ being the noise vector [11,18]. Then the time-dependent inhomogeneous equations of motion arise as $\dot{u}(t) = A(t)u(t) + n(t)$, with

$$A(t) = \begin{bmatrix} 0 & \omega_m & 0 & 0 \\ -\omega_m & -\gamma_m & G_x(t) & G_y(t) \\ -G_y(t) & 0 & -\kappa & \Delta(t) \\ G_x(t) & 0 & -\Delta(t) & -\kappa \end{bmatrix}, \tag{5}$$

where the real $A(t)$ contains the time-modulated coupling constants and the detuning as $G(t) = G_x(t) + iG_y(t)$,

$$G(t) = \sqrt{2}\langle a(t)\rangle G_0, \qquad \Delta(t) = \Delta_0 - G_0\langle q(t)\rangle. \tag{6}$$

From now on we will consider quasiperiodic orbits only, so the long-time dynamics following the initial one, when the first moments follow a motion that is τ periodic. Then, A is

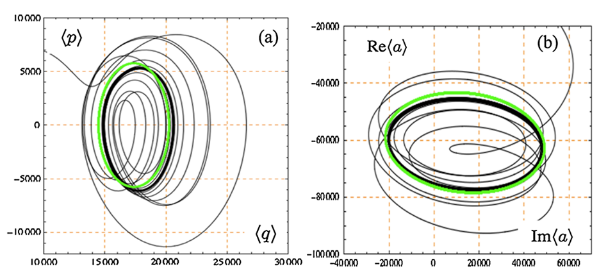

FIG. 1 (color online). Phase space trajectories of the first moments of the mirror (a) and light (b) modes. Numerical simulations for $t \in [0, 50\tau]$ (black) and analytical approximations of the asymptotic orbits (green or light gray).