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Advective flow of permeable sphere in an electrical field

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ABSTRACT

Advective flow of a permeable sphere in an electrical field is comprehensively studied. The sphere has a uniform permeability and is subject to an incoming Newtonian flow. The electrical field generates an electro-osmotic flow inside the sphere, which markedly affects sphere flow dynamics. A numerical model elucidates the effects of flow dynamic parameters on the drag coefficient and ratio of drag forces to a permeable and solid sphere. The model solves the Navier–Stokes equations both inside and outside the porous sphere. The unique flow field and pressure patterns of the permeable sphere flow are characterized in detail, and utilized to interpret the distinguishing flow behaviors of spheres induced by electro-osmotic flow. Drag force decreases and or reverses in direction when the intensity of the electro-osmotic flow in the sphere increases. When the electro-osmotic flow is counter to the incoming flow, drag force increases significantly, and vortices form near the sphere. As the sphere becomes highly permeable, the influence of the electro-osmotic flow and incoming flow velocity are reduced markedly.

1. Introduction

Permeable sphere flow is of considerable importance to many applications, including sedimentation, electro-osmotic dewatering and aggregation of wastewater sludge. The motion of a permeable sphere in an infinite Newtonian fluid depends on the hydrodynamic force on the sphere [1,2]. Since fluid can penetrate a permeable sphere, the corresponding flow resistance for the permeable sphere is lower than that for a solid sphere. Therefore, permeability determines the hydrodynamic characteristics of a permeable sphere.

Naturally formed porous materials, such as wastewater sludge flocs, have complex internal structures [3–5] and, thus, have intrafloc flows [6–12]. Large pores, such as those defined by an image-based method [13], have low flow resistance and, therefore, contribute significantly to the permeability of a porous material, such as a porous fouling layer on a membrane [14]. However, some pores, including large pores, may not connect to the outside of the sphere and thereby cannot contribute to overall permeability. Simplified models assume uniform permeability [15–18] or a radially varying permeability of a sphere [19–23].

How a spherical flow is influenced by an electrical field is of a great concern for many applications, such as the electrophoresis and electro-osmosis of sludge. A fluid flow driven by an electrical field in a porous medium differs from that driven by a pressure gradient [24]. The electrical permittivity of a porous material likely differs from that of surrounding fluids, thereby distorting the electrical lines of force on internal flow fields [25]. The presence of a pressure gradient can also interact with the electrical field [26]. The fluid flow field around and inside a porous sphere under an electrical field has not been well characterized.

This work numerically elucidates the flow fields and drag forces for a porous sphere moving at different Reynolds numbers through an unbound Newtonian fluid under an external electrical field. The sphere has uniform permeability, and the effects of the external electrical field induce electro-osmotic flow.

2. Materials and methods

2.1. Governing equations

The problem of interest is a permeable sphere moving at a constant velocity, u_0, in a quiescent fluid. The sphere is located at the center of a large cylinder filled with a Newtonian fluid moving in the direction along the axis of the cylinder, which comprises the computational domain. A uniform electrical field is applied along/against the moving direction of the sphere and causes an electro-osmotic flow inside a sphere with a porous structure. This problem is hydrodynamically equivalent to that of a fixed sphere experiencing an incoming flow at a velocity of u_0. The computational domain is situated in an axisymmetrical X–R coordinate (Fig. 1).

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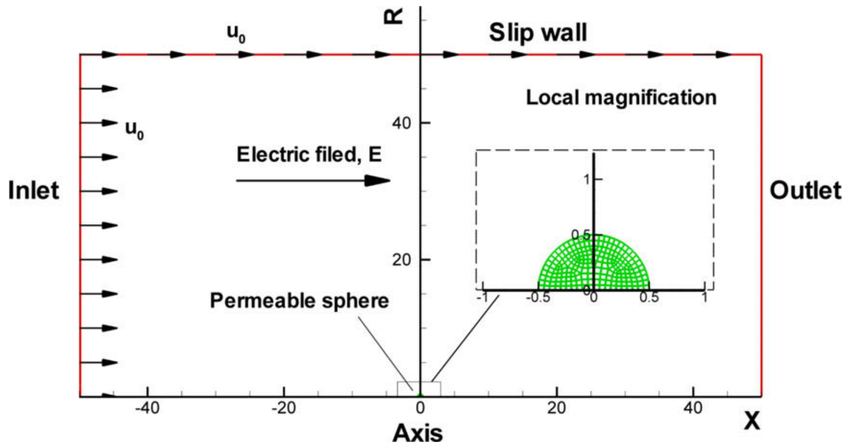

Fig. 1. Computational domain in an axisymmetric coordinate.

Preliminary calculations indicate that when cylinder diameter is 100 times that of the sphere, the wall confinement effect on sphere flow can be neglected and the sphere behaves as if it were moving in an unbound fluid.

The non-dimensional equations for a sphere moving steadily in an unbound incompressible Newtonian fluid are as follows:

(a) Continuity equation:

$$\tilde{\nabla} \cdot \mathbf{U} = 0 \tag{1}$$

(b) Momentum equations:

Inside the sphere:

$$Re\tilde{\nabla} \cdot (\mathbf{UU}) = -\tilde{\nabla}P + \tilde{\nabla} \cdot \mathbf{T} - 4\beta^2\mathbf{U} + 4\beta^2\Psi\mathbf{e_E} \tag{2}$$

Outside the sphere:

$$Re\tilde{\nabla} \cdot (\mathbf{UU}) = -\tilde{\nabla}P + \tilde{\nabla} \cdot \mathbf{T} \tag{3}$$

where $\mathbf{U} = \frac{\mathbf{u}}{u_0}$, $\mathbf{u} = u_x\mathbf{e_x} + u_r\mathbf{e_r}$, $\tilde{\nabla} = \mathbf{e_x}\frac{\partial}{\partial X} + \frac{\mathbf{e}_\theta}{R}\frac{\partial}{\partial \theta} + \mathbf{e_r}\frac{\partial}{\partial R}$, $X = \frac{x}{d}$, $R = \frac{r}{d}$, $\mathrm{T} = [\tilde{\nabla}\mathbf{U} + (\tilde{\nabla}\mathbf{U})^T]$, $P = \frac{pd}{\mu u_0}$, $\beta = \frac{d}{2\sqrt{K_p}}$, $Re = \frac{\rho u_0 d}{\mu}$, and $\Psi = \frac{u_E}{u_0} = \frac{K_E E}{u_0 \mu}$.where $\mathbf{u}$ is the velocity vector; u_x and u_r are components of $\mathbf{u}$ in the X- and R-directions, respectively; $\mathbf{e_x}$ is the unit vector in the axial direction $\mathbf{e}_\theta$ in the angular direction and $\mathbf{e_r}$ in the radial direction; d is the sphere diameter; x, θ, and r the dimensional coordinates in the axial, angular, and radial directions, respectively; p is pressure, ρ is density, and μ is viscosity of a fluid; K_p is the permeability of a pure hydrodynamic flow; and K_E is the electro-osmotic permeability defined by [27] as

$$K_E = \frac{\mu}{E}u_E \tag{4}$$

where E is the electrical field intensity along the flow direction, and u_E is the electro-osmotic flow superficial velocity in a porous medium.

Non-dimensional parameter β is a ratio between sphere radius and a characteristic length based on the permeability of purely hydrodynamic flow. Since the radius of a sphere alters the outflow and permeability introduces the internal flow, β determines the outside and inside flows. Non-dimensional parameter Ψ measures the strength of an electro-osmotic flow relative to the hydrodynamic flow inside the sphere; thus, in this work, is called the *velocity ratio*. At $\Psi = 0$, sphere flow is not subject to an electrical field and becomes a purely hydrodynamic flow.

2.2. Boundary conditions

At the inlet of the computational domain, we assume a uniform flow boundary condition:

$$\mathbf{U} = \mathbf{e_x} \tag{5}$$

Since the distance to the sphere exceeds 50 times its diameter, we assume the outlet boundary condition is

$$\frac{\partial \mathbf{U}}{\partial X} = 0 \tag{6}$$

At the side boundary (50 times the sphere diameter away from the sphere), fluid flow is undisturbed by the presence of the sphere; that is,

$$\mathbf{U} = \mathbf{e_x} \tag{7}$$

On the axisymmetrical axis:

$$\frac{\partial \mathbf{U}}{\partial R} = 0 \tag{8}$$

On the sphere surface:

$$(-P\mathbf{I} + \mathbf{T})|_+ = (-P\mathbf{I} + \mathbf{T})|_- \tag{9a}$$

and

$$\mathbf{U}|_+ = \mathbf{U}|_- \tag{9b}$$

where $\mathbf{I}$ is the identity matrix, and + and − represent the inside and outside of the sphere surface, respectively. Eqs. (9a) and (9b) derive the balance of force and mass flux on the surface, respectively.

2.3. Solution and validation

The computational domain is discretized into finite volumes. All variables are stored in the center of the mesh cells. Convective and diffusive fluxes in the governing equations are discretized by a second-order upwind scheme and a central-differencing scheme, respectively. Pressure–velocity coupling is implemented by the SIMPLE algorithm. A uniform flow condition, as that in Eq. (5), is utilized to initialize the solution. Iterations at each time step are terminated when dimensionless residuals for all equations are $<10^{-6}$. The computations are performed using the commercial software FLUENT 6.1 [28]. Grid dependence is assessed by inspecting the cases under various grid densities. Finally, the grid set with $\Delta X = \Delta R = 0.01$ in the sphere and its vicinity, and $\Delta X = \Delta R = 0.1$ in the faraway region shows a <1% difference in the drag force of that with $\Delta X = \Delta R = 0.005$ (near the sphere) and $\Delta X = \Delta R = 0.05$ (far from the sphere), indicating that a weak dependence on grid size has been achieved. The former grid set is therefore chosen for further analysis.

Model validation is first performed for solid sphere flows. The drag force calculated by the model has a deviation <1% from the prediction by Stokes law for creeping flows with small Re values ($Re < 0.1$). Further comparison between the model and a theoretical