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Dynamic electrophoresis of a spherical dispersion of soft particles subject to a stress-jump condition

Wei-Lun Min, Eric Lee, Jyh-Ping Hsu [*]

Department of Chemical Engineering, National Taiwan University, Taipei 10617, Taiwan

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Abstract

The dynamic mobility of a spherical dispersion of soft particles, where a particle comprises a rigid core and a membrane layer, is evaluated for the case when the shear stress across the membrane layer–liquid interface is discontinuous, the so-called stress-jump condition. We show that, due to the effect of double-layer deformation, the magnitude of the dynamic mobility of a particle has a local maximum and the corresponding phase angle has a negative (phase lead) local minimum at a low to medium level of the frequency of the applied electric field. This effect becomes insignificant if the frequency of the applied electric field is sufficiently high. The stress-jump condition may lead to a significant influence on the drag, and consequently the mobility of a particle. The degree of influence depends upon the sign of the stress-jump coefficient and the charged conditions of the membrane layer of the particle.

Keywords: Dynamic electrophoresis; Spherical dispersion; Soft particles; Stress-jump condition

1. Introduction

Owing to their potential applications, such as the estimation of the surface potential of a colloidal entity in a concentrated dispersion, electroacoustic phenomena have drawn the attention of colloidal scientists in recent years. These phenomena include two reversal effects: applying a supersonic wave to a colloidal dispersion and measuring the responding vibration potential, and applying an alternating electric field and detecting the responding supersonic wave [1]. Several theoretical attempts have been made in the past decade to model electroacoustic phenomena [2–10]. The key factors considered in these studies include the level of the surface potential of a particle, the thickness of the double layer surrounding a particle, the concentration of particles, and the, types of boundary conditions assumed. Compared with the response of a colloidal dispersion to a static applied electric field, the response of a dynamic applied electric field is much more complicated. For instance, the electrophoretic velocity of a particle may not synchronize with the applied electric field; both phase lag and phase lead may be observed. Also, depending upon the operating conditions, both the magnitude and the phase angle of the electrophoretic velocity of a particle can have a complicated relation with the frequency of the applied electric field.

Previous analyses of electroacoustic phenomena all focused on dispersions of rigid entities. While particles of a rigid nature are common, those of nonrigid nature are also of practical significance. Biocolloids such as biological cells and microorganisms, for instance, belong to this category, in that their surfaces can be penetrable to ionic species. A so-called soft particle [11–14], where a particle comprises a rigid core and an ion-penetrable membrane layer, is often adopted to simulate these types of particles. Available results for the behavior of a soft particle when an alternating electric field is applied are very limited. Under conditions of low particle concentration, thick membrane layer, and thin double layer, Ohshima [15] was able to derive an analytical expression for the dynamic electrophoretic mobility of a soft particle. Lopez-Garcia et al. [16] proposed a numerical procedure to solve the problem of Ohshima; an extra hydrodynamic force acting on the rigid core of a particle was considered. In the mathematical treatment of these analyses, it was assumed that both the fluid velocity

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[*] Corresponding author. Fax: +886 2 23623040.
E-mail address: jphsu@ntu.edu.tw (J.-P. Hsu).

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and its shear stress are continuous across the membrane layer–liquid interface. In a study of the flow field involving a porous medium, Ochoa-Tapia and Whitaker [17,18] were able to show that while the former is appropriate, the latter needs modification. A stress-jump condition, characterized by a stress-jump coefficient, was proposed to describe the shear stress across the porous medium–liquid interface. Their treatment was found to be more convincing than the empirical relation proposed by Beavers and Joseph [19]. Follow-up studies [20–22] also revealed that the introduction of a stress-jump condition for the description of the nature of the shear stress across a porous medium–liquid interface is necessary. Sekhar and Sano [21] suggested that the stress-jump coefficient ranges approximately from -0.9 to 0.9.

In this study the electrophoresis of a spherical dispersion of soft particles, where a particle comprises a rigid core and an ion-penetrable membrane layer, subject to an alternating electric field is analyzed. In particular, the influence of the discontinuity in the shear stress across the membrane layer–liquid interface on the electrophoretic behavior of a dispersion is investigated. The dynamic electrophoretic mobility of a particle characterized by its magnitude and phase angle is evaluated under various conditions.

2. Theory

Let us consider the dynamic electrophoresis of a spherical dispersion of soft particles, where a particle comprises a rigid core of radius a and a membrane layer of thickness d. The dispersion medium contains z_1:z_2 electrolyte; z_1 and z_2 are respectively the valence of cations and that of anions with $z_2 = -\alpha z_1$. Referring to Fig. 1, the unit cell model of Kuwabara [23] is adopted, where the dispersion is simulated by a representative particle and a concentric spherical liquid shell of radius c. Let $b = a + d$ and $H = (b/c)^3$; the latter is a measure for the volume fraction of particles. The spherical coordinates (r, θ, φ) are adopted with origin located at the center of the particle. $\mathbf{E} = E_Z e^{-i\omega t}\mathbf{e}_z$ is the applied electric field, where $\mathbf{e}_z$ is the unit vector in the z-direction, t is the time, ω is the frequency of $\mathbf{E}$, and $i = \sqrt{-1}$. $\mathbf{U} = Ue^{-i\omega t}\mathbf{e}_z = U_R + iU_I$ is the electrophoretic velocity of the particle; U_R and U_I are respectively the real and the imaginary parts of $\mathbf{U}$. For the present case the electric potential ϕ can be described by the Poisson equation

$$\nabla^2\phi = -\frac{\rho + \ell\rho_{\text{fix}}}{\varepsilon}, \tag{1}$$

where $\rho = \sum_j n_j z_j \hat{e}$ and ρ_{fix} are respectively the space density of mobile ions and the fixed-charge density in the membrane layer, ∇ is the gradient operator, $\hat{e}$ and ε are respectively the elementary charge and the permittivity of the liquid phase, n_j and z_j are respectively the concentration and the valence of ionic species j, and ℓ is a region index, $\ell = 1$ for $a < r < b$, and $\ell = 0$ for $b < r < c$. We assume that both ε and ρ_{fix} are constant.

The conservation of the jth ionic species leads to

$$\frac{\partial n_j}{\partial t} = D_j\left[\nabla^2 n_j + \frac{z_j e}{kT}\left(\nabla n_j \cdot \nabla\phi + n_j\nabla^2\phi\right)\right] - \nabla n_j \cdot \mathbf{v}, \tag{2}$$

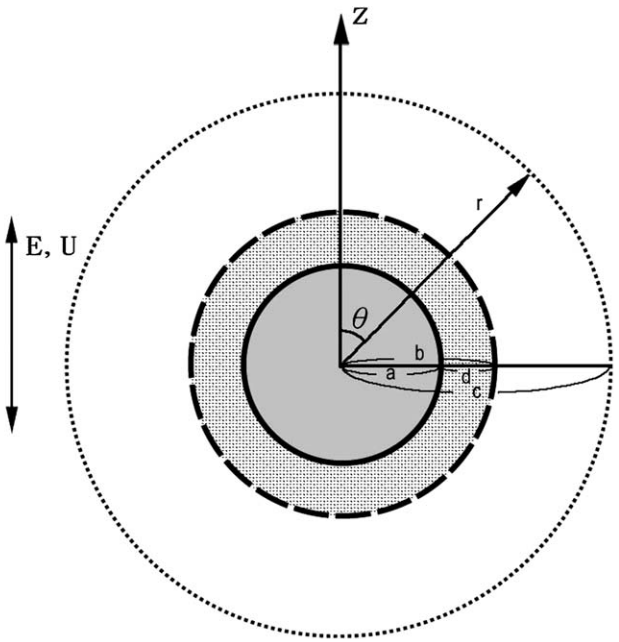

Fig. 1. A spherical dispersion of soft particles, where a particle comprises a rigid core of radius a and a membrane layer of thickness d, is simulated by a unit cell model where a cell comprises a representative particle and a concentric spherical liquid shell of radius c. The spherical coordinates (r, θ, φ) are adopted with origin located at the center of the representative particle. **E** and **U** are respectively the applied electric field and the electrophoretic velocity of a particle.

where D_j is the diffusivity of ionic species j, k is the Boltzmann constant, T is the absolute temperature, and $\mathbf{v}$ is the liquid velocity.

If we assume that the liquid phase is an incompressible Newtonian fluid with constant physical properties, then the flow field can be described by

$$\nabla \cdot \mathbf{v} = 0, \tag{3}$$

$$\rho_f \frac{\partial \mathbf{v}}{\partial t} - \nabla p + \eta\nabla^2\mathbf{v} - \rho\nabla\phi - \ell\gamma\mathbf{v} = 0, \tag{4}$$

where p is the pressure, η and ρ_f are respectively the viscosity and the density of the liquid phase, and γ is a friction coefficient characterizing the flow of liquid in the membrane layer.

For convenience, ϕ, n_j, $\mathbf{v}$, and p are all decomposed into an equilibrium term and a perturbed term. That is,

$$\phi(r, \theta, t) = \phi_e(r) + \delta\phi(r, \theta)E_z e^{-i\omega t}, \tag{5}$$

$$n_j(r, \theta, t) = n_j^e(r, \theta) + \delta n_j(r, \theta)E_z e^{-i\omega t}, \tag{6}$$

$$\mathbf{v}(r, \theta, t) = \mathbf{v}_e + \delta\mathbf{v}(r, \theta)E_z e^{-i\omega t}, \tag{7}$$

$$p(r, \theta, t) = p_e(r, \theta) + \delta p(r, \theta)E_Z e^{-i\omega t}. \tag{8}$$

The subscript and superscript e in these expressions represents equilibrium properties, and δ denotes a perturbed property. Note that since the liquid phase remains stagnant in the absence of **E**, we must have $\nabla p_e = 0$ and $\mathbf{v}_e = \mathbf{0}$; p_e and $\mathbf{v}_e$ are respectively the equilibrium pressure and the equilibrium liquid velocity. Substituting Eqs. (5)–(8) into Eqs. (1)–(4) leads to two subproblems, one for the equilibrium system and the other for the perturbed system.