Available online at www.sciencedirect.com

SCIENCE DIRECT®

Journal of Colloid and Interface Science 300 (2006) 327–337

Journal of Colloid and Interface Science

www.elsevier.com/locate/jcis

# Model for Plateau border drainage of power-law fluid with mobile interface and its application to foam drainage

Zebin Wang, Ganesan Narsimhan *

*Biochemical and Food Process Engineering, Department of Agricultural and Biological Engineering, Purdue University, West Lafayette, IN 47907, USA*

Received 2 December 2005; accepted 11 March 2006

## Abstract

A model for drainage of a power-law fluid through a Plateau border is proposed which accounts for the actual Plateau border geometry and interfacial mobility. The non-dimensionalized Navier–Stokes equations have been solved using finite element method to obtain the contours of velocity within the Plateau border cross section and average Plateau border velocity in terms of dimensionless inverse surface viscosity and power-law rheological parameters. The velocity coefficient, the correction for the average velocity through a Plateau border of actual geometry compared to that for a simplified circular geometry of the same area of cross section, was expressed as a function of dimensionless inverse surface viscosity and flow behavior index of the power-law fluid. The results of this improved model for Plateau border drainage were then incorporated in a previously developed foam drainage model [G. Narsimhan, J. Food Eng. 14 (1991) 139] to predict the evolution of liquid holdup profiles in a standing foam. Foam drainage was found to be slower for actual Plateau border cross section compared to circular geometry and faster for higher interfacial mobility and larger bubble size. Evolution of liquid holdup profiles in a standing foam formed by whipping and stabilized by 0.1% $\beta$-lactoglobulin in the presence of xanthan gum when subjected to $16g$ and $45g$ centrifugal force fields was measured using magnetic resonance imaging for different xanthan gum concentrations. Drainage resulted in the formation of a separate liquid layer at the bottom at longer times. Measured bubble size, surface shear viscosity of $\beta$-lactoglobulin solutions and literature values of power-law parameters of xanthan gum solution were employed in the current model to predict the evolution of liquid holdup profile which compared well with the experimental data. Newtonian model for foam drainage for zero shear viscosity underpredicted drainage rates and did not agree with the experimental data.

*Keywords:* Plateau border drainage; Power-law fluid; Interfacial mobility; Surface shear viscosity; Foam drainage; Liquid holdup profile; Magnetic resonance imaging; Protein stabilized foam; $\beta$-Lactoglobulin; Xanthan gum

## 1. Introduction

A foam is a high volume fraction dispersion of gas in liquid in which the gas bubbles are distorted in the form of polyhedra and are separated by thin films. Three adjacent thin films intersect in a Plateau border and the continuous phase is interconnected through a network of Plateau borders [1]. Liquid drains from thin films to the neighboring Plateau border because of the pressure difference caused by the radius of curvature of Plateau border [2]. The liquid in Plateau border drains through the network of Plateau borders due to gravity thereby resulting in a liquid holdup profile in a standing foam with the bottom of the foam being wetter [3,4]. Since the top of the foam is drier with a smaller area of cross section of Plateau border and hence smaller radius of curvature, a gradient of Plateau border suction develops which opposes gravity [4]. Plateau border suction is counterbalanced by the disjoining pressure due to intermolecular interactions between the two faces of the film [5] at which point the film reaches a mechanical equilibrium.

A comprehensive review of drainage of Newtonian fluids through thin films is given by Ivanov [5]. Previous investigations [6,7] have obtained the velocity of drainage of Newtonian fluid through different simplified geometries of Plateau border from the solution of Navier–Stokes equation for immobile gas–liquid interfaces. This has been extended for mobile gas–liquid interfaces [8,9]. Wang and Narsimhan [10] have recently investigated the flow of a power-law fluid through a simplified

---

* Corresponding author.
*E-mail address:* narsimha@ecn.purdue.edu (G. Narsimhan).

0021-9797/$ – see front matter © 2006 Elsevier Inc. All rights reserved.
doi:10.1016/j.jcis.2006.03.023

circular geometry of a Plateau border with immobile interface from the solution of Navier–Stokes equation. Foam structure was incorporated in a mechanistic model for foam drainage [3,4,11–15] for the prediction of liquid holdup profile in a standing foam. Thin film rupture [16] has been incorporated in foam drainage models to predict the change in bubble size distribution due to coalescence [12] and foam collapse [3]. Experimental measurements of flow through a single Plateau border [17] do not seem to agree with theory and this discrepancy can be attributed to dissipation processes of the interface as well as to the liquid flows induced in adjoining films. Foam drainage in a standing foam was characterized by the accumulation of liquid layer [18] and liquid holdup profiles using magnetic resonance imaging [10,19–21] and compared with model predictions [10,18]. The rate of advance of a moving front at different liquid holdups was measured using light scattering [22–25] when liquid was introduced at the top of a standing foam in order to obtain the steady state foam velocity. Foam velocity was found to satisfy a power-law dependence on the liquid holdup with the value of the exponent being a function of interfacial mobility [23]. Foam drainage through a network of Plateau borders and nodes was viewed as flow through a porous medium with permeability that is dependent on liquid holdup [22,24]. A vertex correction to the foam drainage equation was proposed [26]. Various models for foam structure, drainage and stability are given in comprehensive reviews [2,27]. In a recent paper, we [10] incorporated our proposed model for Plateau border drainage of power-law fluid for a simplified circular geometry with immobile interface into foam drainage equation to predict the evolution of liquid holdup profile in a protein stabilized standing foam and compared the model predictions with experiments for foam stabilized by sodium caseinate. In this paper, the model for Plateau border drainage of a power-law fluid is refined by accounting for the actual Plateau border geometry as well as for interfacial mobility in terms of surface viscosity. This model is then employed in foam drainage to predict the evolution of liquid holdup profile in a standing foam stabilized by $\beta$-lactoglobulin and compared with experimental data obtained using magnetic resonance imaging.

## 2. Materials and methods

### 2.1. Sample procedure

Foams were prepared by whipping air into 100 ml of 0.1 to 0.3 wt% $\beta$-lactoglobulin (Sigma Cat. No. 0130) solutions using a Sunbeam Mixmaster mixer for 1 h. Speed level 12 was applied. Different concentrations (0.1 and 0.3 wt%) of xanthan gum (Sigma, G1253) were added to increase viscosity. Foam was placed into a cylindrical sample cell (13 mm ID, 20 mm deep) and centrifuged in an Allegra 21R centrifuge (Beckman coulter) for 10–60 min at 16$g$ or 45$g$.

### 2.2. Magnetic resonance imagining

The variation of liquid holdup with height was measured immediately after centrifugation using magnetic resonance imaging (MRI) [20]. The sample cell consisting of foam was inserted into a 20 MHz MARAN Ultra Magnetic imaging spectrometer manufactured by Resonance Instruments Ltd., Witney, UK. The sample was subjected to a magnetic field gradient along the vertical axis. Sample was scanned continuously to produce 1 H spectra at 15 MHz. Four scans were performed for each profile, which took about 1 min. The foam was so stable that the sample can be assumed to change very little during sampling. The liquid holdup at different vertical locations within the foam sample was inferred from the experimental measurement of signal intensity. The details of this measurement are given elsewhere [10]. The interval between 90° and 180° pulses and pre-delay between successive signal acquisitions were set at 3 ms and 15 s, respectively, as the program parameters.

### 2.3. Bubble size measurement

Small samples were placed between glass slides under a light microscope (Spencer) with an amplification of about 80×. Photographs were taken using Polaroid instant pack films. Bubble sizes were measured from the photographs to obtain the bubble size distribution. Two to three hundred bubbles were observed for each distribution.

### 2.4. Surface shear viscosity and elasticity

Surface shear viscosity and elasticity of 0.1 wt% $\beta$-lactoglobulin were measured using the Camtel CIR-100 Rheometer (Camtel Ltd., UK). A platinum ring was placed at the air–protein solution interface. A fixed torque was applied to the ring at 0.1 Hz and the resulting strain was measured to obtain surface elasticity and viscosity. The details of the instrument are described elsewhere [28]. Measurements were made after allowing $\beta$-lactoglobulin to adsorb for 3 h in order to allow the interface to reach equilibrium.

## 3. Drainage in actual Plateau border geometry

Cross section of a Plateau border is shown in Fig. 1. Because of symmetry, only 1/6th of the Plateau border cross section needs to be solved for the velocity profile. The film thickness is usually negligibly small compared to the Plateau border cross section. Therefore, we neglect the film thickness in order to

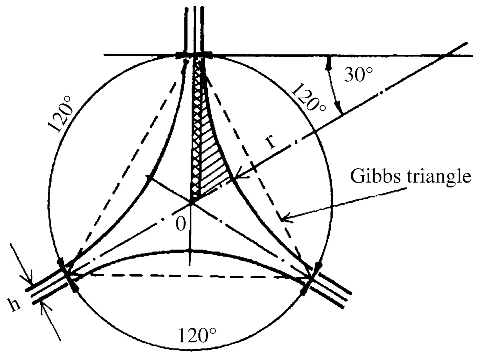

Fig. 1. Actual cross section of a Plateau border.