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Influence of viscous dissipation and thermophoresis on Darcy–Forchheimer mixed convection in a fluid saturated porous media

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Abstract

Mixed convection flow, heat, and mass transfer about an isothermal vertical flat plate embedded in a fluid-saturated porous medium and the effects of viscous dissipation and thermophoresis in both aiding and opposing flows are analyzed. The similarity solution is used to transform the problem under consideration into a boundary value problem of coupled ordinary differential equations, which are solved numerically by using the shooting method. Numerical computations are carried out for the nondimensional physical parameter. The results are analyzed for the effect of different physical parameters such as thermophoretic, mixed convection, inertia parameter, buoyancy ratio, and Schmid number on the flow, heat, and mass transfer characteristics. Two cases are considered, one corresponding to the presence of viscous dissipation and the other to the absence of it.

Keywords: Heat and mass transfer; Viscous dissipation; Porous media; Mixed convection; Thermophoresis

1. Introduction

Natural convective flow and heat transfer in saturated porous media is gaining more attention because of its wide applicability in packed beds, porous insulation, beds of fossil fuels, nuclear waste disposal, resin transfer modeling, etc. Over the past two decades, studies in aerosol particle deposition due to thermophoresis have gained importance for engineering applications. The technological problems include particle deposition onto wafers in the microelectronics industry, particle surfaces produced by condensing vapor–gas mixtures, particles impacting the blade surface of gas turbines, and others such as filtration in gas cleaning and nuclear reactor safety. In engineering particle, usually more than one mechanism can act simultaneously and their interactions need to be considered for accurate prediction of deposition rates. In this work, the mechanism of particle deposition onto a vertical surface by the coupled effects of viscous dissipation, mixed convection, and thermophoresis is examined.

Most of the research efforts [1–4] concerned free convection using Darcy's law, which states the volume-averaged velocity is proportional to the pressure gradient. The Darcy model is shown to be valid under conditions of low velocities and small porosity. In many practical situations, the porous medium is bounded by an impermeable wall, has high flow rates, and reveals nonuniform porosity distribution in the near-wall region, thereby making Darcy's law inapplicable. To model the real physical situations better, it is therefore necessary to include the non-Darcian effects in the analysis of convective transport in a porous medium. The problem of Darcy–Forchheimer mixed convection heat and mass transfer in fluid-saturated porous media was studied by Rami et al. [5]. Seddeek [6] studied the effects of magnetic field, variable viscosity, and non-Darcy effects on forced convection flow about a flat plate with variable wall temperature in porous media. Recently, Seddeek [7] studied non-Darcian effects on forced-convection heat transfer over a flat plate in a porous medium.

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Goren [8] was one of the first to study the role of thermophoresis in the laminar flow of a viscous and incompressible fluid. He used the classical problem of flow over a flat plate to calculate deposition rates and showed that substantial changes in surface deposition can be obtained by increasing the difference between the surface and free stream temperatures. This was later followed by the effect of thermophoresis on particle deposition from a mixed convection flow onto a vertical plate by Chang et al. [9] and Jayaraj et al. [10]. Also, Tsai [11] obtained the effect of wall suction and thermophoresis on aerosol particle deposition from a laminar flow over a flat plate. Selim et al. [12] discussed the effect of surface mass transfer on mixed-convection flow past a heated vertical permeable flat plate with thermophoresis. Recently, Chamkha and Pop [13] studied the effect of thermophoretic particle deposition in free convection boundary layer from a vertical flat plate embedded in a porous medium. Peev et al. [14] discussed the problem of heat transfer from solid particles to power low non-Newtonian fluid in a granular bed at low Reynolds number.

Most previous studies of the same problem neglected viscous dissipation and thermophoresis. But Gebhart [15] has shown that the viscous dissipation effect plays an important role in natural convection in various devices that are subjected to large variations of gravitational force or that operate at high rotational speeds. Motivated by the above investigations and possible applications, it is of interest in the present work to study viscous dissipation and thermophoresis in Darcy–Forchheimer mixed convection heat and mass transfer in fluid-saturated porous media.

2. Mathematical formulation

Consider the steady mixed convection boundary layer over a vertical flat plate of constant temperature T_w and concentration C_w, which is embedded in a fluid-saturated porous medium of ambient temperature T_∞ and concentration C_∞, respectively. The x-coordinate is measured along the plate from its leading edge and the y-coordinate normal to it. Allowing for both Brownian motion of particles and thermophoretic transport, the governing boundary layer equations are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\frac{\partial u}{\partial y}+\frac{c_\mathrm{f}\sqrt{K_1}}{\nu}\frac{\partial (u^2)}{\partial y}=\pm g\frac{K_1}{\nu}\left(\beta_\mathrm{T}\frac{\partial T}{\partial y}+\beta_\mathrm{c}\frac{\partial C}{\partial y}\right), \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\lambda_\mathrm{g}}{\rho c_\mathrm{p}}\frac{\partial^2 T}{\partial T^2}+\frac{\nu}{c_\mathrm{p}}\left(\frac{\partial u}{\partial y}\right)^2, \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-\frac{\partial}{\partial y}(V_\mathrm{T}C). \tag{4}$$

The boundary conditions are given by

$$\begin{aligned}&y=0:\quad v=0,\quad T=T_\mathrm{w},\quad C=C_\mathrm{w},\\&y\to\infty:\quad u=u_\infty,\quad T=T_\infty,\quad C=C_\infty,\end{aligned} \tag{5}$$

where u, v are velocity components along x, y coordinates, respectively, T and C are, respectively, the temperature and concentration, c_f is the Forchheimer coefficient, K_1 is the Darcy permeability, g is the acceleration due to gravity, ν is the kinematic viscosity, β_T is the coefficient of thermal expansion, β_c is the coefficient of concentration expansion, c_p is the specific heat of the fluid at constant pressure, q_r is the radiative heat flux, and D is the mass diffusivity. In Eq. (2), the plus sign corresponds to the case where the buoyancy force has a component "aiding" the forced flow and the minus signs refer to the "opposing" case.

In Eq. (4), the thermophoretic velocity V_T was given by [16]

$$V_\mathrm{T}=-k\nu\frac{\nabla T}{T}=-\frac{k\nu}{T}\frac{\partial T}{\partial y}, \tag{6}$$

where k is the thermophoretic coefficient, which is given by [17] as

$$k=\frac{2C_\mathrm{s}(\lambda_\mathrm{g}/\lambda_\mathrm{p}+C_\mathrm{t}\,\mathrm{Kn})C_1}{(1+3C_\mathrm{m}\,\mathrm{Kn})(1+2\lambda_\mathrm{g}/\lambda_\mathrm{p}+2C_\mathrm{t}\,\mathrm{Kn})}, \tag{7}$$

where C_m, C_s, and C_t are constants and λ_g and λ_p are the thermal conductivities of the fluid and diffused particles, respectively. C_1 is the Cunningham correction factor and Kn is the Knudsen number.

Now we define the following dimensionless variables for mixed convection:

$$\begin{aligned}&\eta=\mathrm{Pe}_x^{1/2}\frac{y}{x},\qquad \psi=\alpha\,\mathrm{Pe}_x^{1/2}f(\eta),\\&\theta(\eta)=(T-T_\infty)/(T_\mathrm{w}-T_\infty),\\&\phi(\eta)=(C-C_\mathrm{w})/(C_\mathrm{w}-C_\infty),\end{aligned} \tag{8}$$

where ψ is the stream function that satisfies the continuity equation and η is the dimensionless similarity variable. With these changes of variables, Eq. (1) is identically satisfied and Eqs. (2)–(4) are transformed to

$$f''+2\Lambda f''f'=\pm\left(\frac{\mathrm{Ra}_x}{\mathrm{Pe}_x}\right)(\theta'+N\phi'), \tag{9}$$

$$\theta''+\frac{1}{2}f\theta'+\mathrm{Pr}\,\mathrm{Ec}\,f''^2=0, \tag{10}$$

$$\frac{1}{\mathrm{Sc}}\phi''-\tau(\varphi\theta''+\varphi'\theta')+\frac{1}{2\,\mathrm{Pr}}f\phi'=0. \tag{11}$$

The corresponding boundary conditions take the form

$$\begin{aligned}&f(0)=0,\quad \theta(0)=1,\quad \phi(0)=1,\\&f'(\infty)=1,\quad \theta(\infty)=0,\quad \phi(\infty)=0,\end{aligned} \tag{12}$$

where the primes denote differentiation with respect to η, $\Lambda=c_\mathrm{f}\sqrt{K_1}u_\infty/\nu$ is the inertia parameter, $\mathrm{Ra}_x=(K_1 g\beta_\mathrm{T}\times(T_\mathrm{w}-T_\infty)x/\alpha\nu$ is the thermal Rayleigh number, $\mathrm{Pe}_x=u_\infty x/\alpha$ is the local Peclet number, $N=\beta_\mathrm{c}(C_\mathrm{w}-C_\infty)/\beta_\mathrm{T}(T_\mathrm{w}-T_\infty)$ is the buoyancy ratio, $\tau=-K(T_\mathrm{w}-T_\infty)/T$