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Sedimentation of a circular disk in power law fluids

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Abstract

The continuity and momentum equations have been solved numerically for the two-dimensional steady flow of power law fluids over a thin circular disk oriented normal to the direction of flow. Extensive results on the individual and total drag coefficients are obtained as functions of the power law flow behavior index ($0.4 \leqslant n \leqslant 1.0$), Reynolds number ($1 \leqslant \mathrm{Re} \leqslant 100$) and the blockage ratio, disk-to-cylinder diameter ratio ($0.02 \leqslant e \leqslant 0.5$), which can be used to estimate the settling velocity of a circular disk. The numerical predictions of drag are consistent with the scant experimental results available in the literature.

Keywords: Disk; Drag; Wall effects; Power law liquids; Sedimentation velocity

1. Introduction

Over the past fifty years or so, much has been written about the drag and wall effects for a sphere translating in quiescent power law liquids [1–3]. While the pertinent literature is nowhere near as extensive as that for spheres falling in Newtonian liquids [4], it suffices to say here that reliable numerical values of drag for the unconfined flow of power law fluids over a sphere are now available up to the sphere Reynolds number of 100 or so, for both shear thinning and shear thickening fluid behavior conditions [5–7], and these are complemented by experimental results which encompass values of the sphere Reynolds number up to about ∼1500 [8–10]. It is readily acknowledged that non-spherical particles are encountered much more frequently in industrial applications than the spherical particles. While a sizable body of information is available on the drag behavior of regular shaped non-spherical particles in Newtonian fluids [4,11,12], very little is known about the drag on non-spherical particles in power law fluids [13–15]. In particular, very limited information is available on the drag behavior of thin disks and plates as encountered in the handling of kaolin clay suspensions used for coating applications. Notwithstanding the fact that the sedimentation behavior of such suspensions of plate-like particles (kaolin, mica, quartz, etc.) is also influenced by several other factors such as aggregation, particle–particle interaction, non-uniform flow conditions and size distribution, etc., it seems appropriate to begin with the simplest case of a single disk with well-defined orientation, which, in turn, can be used to build up the level of complexity in a gradual manner. Hence, this work presents the results of a numerical study on the drag and wall effects for a thin circular disk sedimenting broad-face-wise in a long cylindrical tube filled with a power law fluid. However, prior to presenting the new results obtained in this study, it is instructive to review briefly the scant literature available in this field, especially relating to the sedimentation of thin disks and plates in power law liquids, whereas the corresponding body of knowledge for disks falling in Newtonian fluids has been recently summarized elsewhere [16].

2. Previous work

A cursory inspection of the available literature on the drag on non-spherical particles in free-settling conditions [13,14,17–20] indicates that indeed very little is known about the drag and wall effects for circular disks in power law fluids. Thus, for instance, Peden and Luo [21] and Reynolds and Jones [22] have reported limited results on the free settling velocity of disks and marble chips in power law liquids. Peden and Luo [21]

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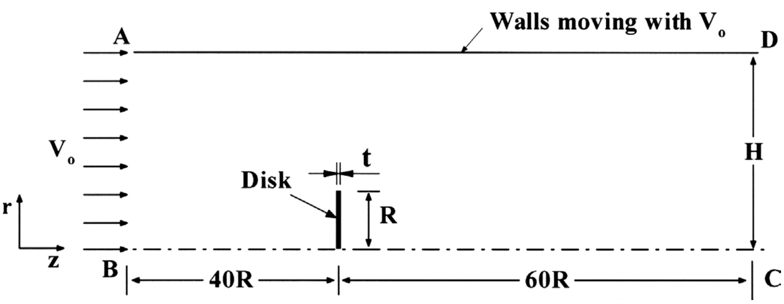

Fig. 1. Schematics of flow.

used only two disks with thickness-to-diameter ratio of ∼0.33 (of sphericity values of 0.75 and 0.76) and correlated their results by empirically modifying the Stokes drag by introducing a function of particle sphericity. Unfortunately, the fact that their experimental results for spheres in Newtonian liquids deviate from the literature values by up to 30% casts some doubt about the accuracy of their results for disks. This uncertainty is further accentuated by the fact that their results obtained with chemically different polymers failed to collapse onto one curve even when these solutions have identical values of power law index thereby severely limiting the prediction capability of their expression. While the reasons for this behavior are not immediately obvious, the possible viscoelastic effects cannot be ruled out. Furthermore, these results relate only to the two values of power law index of 0.42 and 0.6 and the maximum value of the Reynolds number (based on equal volume sphere) is about 50. Reynolds and Jones [22], on the other hand, simply demonstrated that in the creeping (low Reynolds number) flow region, the disk-like and sphere-like marble chips of equal volume settled with comparable terminal velocity. This qualitative inference is also in line with the subsequent experimental findings of Chhabra et al. [23] and Rami et al. [24]. Both Chhabra et al. [23] and Rami et al. [24] concluded that the drag coefficient of disks in power law fluids was in line with the corresponding expression for drag in Newtonian liquids [4,16,25]. Combined, these results embrace the following ranges of conditions: $Re \leqslant 100$ and power law index from 0.48 to 1. However, in both these studies, the disks settled with broad-face normal to the direction of gravity and the Reynolds number was defined using the disk diameter and not the equal volume sphere diameter, as was the case with the study of Peden and Luo [21]. The only other pertinent study is that of Chhabra [19] in which the wall effects on a freely falling disks were studied, and these were found to be less severe than that in Newtonian liquids, a trend which is also consistent with that for a spherical particle settling at the axis of a cylinder. However, this study was limited to a maximum value of the Reynolds number of 7.

In summary, it is therefore safe to conclude that no prior theoretical/numerical results are available on the influence of power law rheology on drag and on wall effects for a freely falling disk in power law fluids. Reported in this study are extensive numerical results on the drag coefficient for a circular disk over wide ranges of rheological parameters, of Reynolds number and of disk-to-cylinder diameter ratio. The paper is concluded by presenting comparisons with the scant experimental results available in the literature.

3. Problem description and governing equations

For the steady and incompressible two-dimensional flow of a power law liquid over a circular disk (of radius R and of negligible inertia) placed concentrically in a cylindrical tube of radius H, $V_\theta = 0$ and no flow variable depends on θ-coordinate (Fig. 1). The fluid enters the tube with a uniform velocity V_0 and the tube walls also move with the same velocity. Hence the velocity profile always remains flat across the tube cross section, except in the vicinity of the disk. The two-dimensional and axisymmetric flow is governed by the equations of continuity and of momentum written as follows:

Continuity equation

$$\frac{1}{r}\frac{\partial}{\partial r}(rV_r) + \frac{\partial V_z}{\partial z} = 0. \tag{1}$$

r-component of momentum equation

$$\rho\frac{DV_r}{Dt} = -\frac{\partial p}{\partial r} + \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rr}) + \frac{\partial \tau_{rz}}{\partial z}. \tag{2a}$$

z-component of momentum equation

$$\rho\frac{DV_z}{Dt} = -\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) + \frac{\partial \tau_{zz}}{\partial z}. \tag{2b}$$

The extra stress components for a power law fluid are given by

$$\tau_{ij} = 2\eta\varepsilon_{ij}, \tag{3}$$

where the power law viscosity η is given by

$$\eta = m(I_2/2)^{(n-1)/2}. \tag{4}$$

The expressions for the components of the rate of deformation tensor, ε_{ij}, in cylindrical coordinates and that for I_2, the second invariant of the rate of deformation tensor, are available in standard texts, e.g., Bird et al. [26].

The physically admissible boundary conditions for this problem are that of no-slip ($V_r = 0$, $V_\theta = 0$, $V_z = 0$) on the surface of the disk including the lateral surface and on the solid boundary walls, i.e., at the walls, AD, $V_z = V_0$ and $V_r = 0$ are implemented. In addition to these, at the exit plane, DC (Fig. 1), the so-called zero diffusion flux condition for all flow variables was specified. This is qualitatively similar to the well-known Orlanski condition [27] used extensively in numerical studies for such flow problems.