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Closed-form analytical solutions for calculation of loads and contact pressures for roller and ball bearings

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ABSTRACT

Closed-form load distribution solutions for calculation of the loads exerted by the rolling elements on the outer raceway in cylindrical and spherical roller bearings under radial loading are proposed. The analytical solutions are derived from the non-conforming contact solution of Hertz and the conforming contact solution of Persson. The maximum pressures due to the maximum loads of the rolling elements calculated from the analytical solutions are within a few percents of the results from the corresponding two-dimensional and three-dimensional finite element analyses. A simple method to estimate the fatigue lives of bearings based on the analytical solutions is also presented for engineers to compare the fatigue performance of bearings with different designs.

1. Introduction

Bearings are essential components in machines and vehicles allowing relative motions of components with minimal friction. The fatigue lives of bearings in machines and vehicles are expected to outlast the lives of the machines and vehicles themselves. However, large cyclic service loads will tend to shorten the lives of the bearings. For use in vehicles, bearings should be designed to withstand both the constant load due to the vehicle weights and the service loads caused by the operation of vehicles.

A bearing should be designed such that the mechanical stresses due to these loads are under some design limits, with the condition that the bearing is maintained under well lubricated conditions. Therefore, the mechanical stresses within the bearing components should be well studied during the design and selection process to optimize the fatigue life of a bearing.

In order to test the structural durability and the failure lives of bearings such as those in vehicles, the bearings are usually put in a test equipment and rotated while external loads are applied. The Society of Automotive Engineers has a standardized test for automotive biaxial wheel tests which is well documented by Nurkala and Wallace [1]. However, extensive testing time on the test equipment is necessary to duplicate the failure modes of bearings under actual operational conditions. Finite element analyses may be considered as a reasonable alternative but have their limitations. Solving multiple and simultaneous contact problems using finite element analyses, as is the case for the bearing analysis, requires a tremendous amount of pre-processing time and computing power which again creates time and cost issues.

In order to overcome these time and cost related issues, an analytical solution is proposed here to estimate the loads exerted by the rolling elements on the raceway in a bearing. Stribeck [2] proposed an equation to find the maximum load on a rolling element based on experimental observations. Goodelle et al. [3] developed a method to determine the static load distribution for radial as well as thrust rolling elements. Harris [4] developed methods to obtain the load distribution based on radial and thrust integrals.

Once the load distribution or the loads exerted by rolling elements are obtained, the contact pressures and the subsurface contact stresses can be obtained analytically from the solutions by McEwen as listed in [5] or Sackfield and Hills [6]. The subsurface stresses can be combined with the existing multiaxial fatigue theories and the critical plane approach such as those by Findley [7] or Socie [8] or standardized bearing life calculation relationships such as those by Lundberg and Palmgren as discussed in [9], Ioannides and Harris [10] and Lösche [11].

The proposed analytical solution which is based on the previous well known analytical solutions by Hertz [5] and Persson [12] is efficient since it is a closed-form solution which does not require trial-and-error iterations. By using the proposed analytical solution, the same bearing evaluation procedure which requires significant CPU time by the corresponding finite element analysis can be accomplished quickly and then a preliminary assessment of a given bearing design can be provided. Also, certain geometric parameters such as the inner and outer raceway diameters, the

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diameter/number of the rolling elements, and the applied load can be adjusted to decide the design combination that can give the optimum fatigue life before initiating costly experiments or computations.

In this paper, the elastic contact theory by Hertz for cylindrical contact is first reviewed. The analytical work by Persson for conforming contact is also reviewed. Based on the solutions by Hertz and Persson, analytical load distribution solutions are derived to calculate the distributions of the loads of the cylindrical and spherical rolling elements in bearings. Two-dimensional and three-dimensional finite element analyses were conducted to validate the accuracy of the proposed analytical solutions. The limitations of the analytical solutions are also discussed. Finally, conclusions are made.

2. Closed-form solutions of Hertz and Persson

2.1. Hz solution for non-conforming contact

The solution for the contact between elastic cylindrical bodies was derived by Hertz [5]. As schematically shown in Fig. 1(a), a segment of an infinitely long cylinder with the radius R is pressed onto a segment of the flat surface of a semi-infinite solid by a load per unit length, P. The Cartesian X, Y and Z coordinates are shown in the figure. The cylinder makes contact with the flat surface over a long strip of area with a width of $2a$ in the Y direction as shown in the figure. Here, a is defined as the half contact width. Due to the contact, an elliptical contact pressure profile $p(x)$ is created on the flat surface along the long strip of area.

For the cylinder pressed in contact with the flat surface by a load per unit length P as in Fig. 1(a), the contact pressure profile $p(x)$ can be expressed as

$$p(x) = \frac{2P}{\pi a^2}\left(a^2 - x^2\right)^{1/2}, \quad -a \le x \le a \tag{1}$$

where x denotes the coordinate in the X direction. Fig. 1(b) shows the normalized contact pressure $p(x)/p_{\max}$ as a function of the normalized distance from the symmetry plane of contact, x/a. The relationship between the load per unit length P and the half contact width a is given by

$$a = \sqrt{\frac{4PR}{\pi E^*}} \tag{2}$$

where E^* is the equivalent elastic modulus defined as

$$\frac{1}{E^*} = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}. \tag{3}$$

Here, E_1, ν_1, E_2 and ν_2 are the elastic moduli and the Poisson's ratios for the cylinder and the semi-infinite solid, respectively. When two elastic cylinders in contact with each other as shown in Fig. 1(c), the contact pressure profile $p(x)$ and the half contact width still follow Eqs. (1) and (2), respectively. However, R becomes the relative radius of curvature defined by the radii of curvature of the two contacting cylinders, R_1 and R_2, as

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}. \tag{4}$$

For the cylinder and flat surface problem, R is equal to R_1 when the radius of curvature of the flat surface, R_2, becomes infinity. The maximum contact pressure can be found by substituting $x = 0$ into Eq. (1) and combining with Eq. (2) as

$$p_{\max} = \sqrt{\frac{PE^*}{\pi R}}. \tag{5}$$

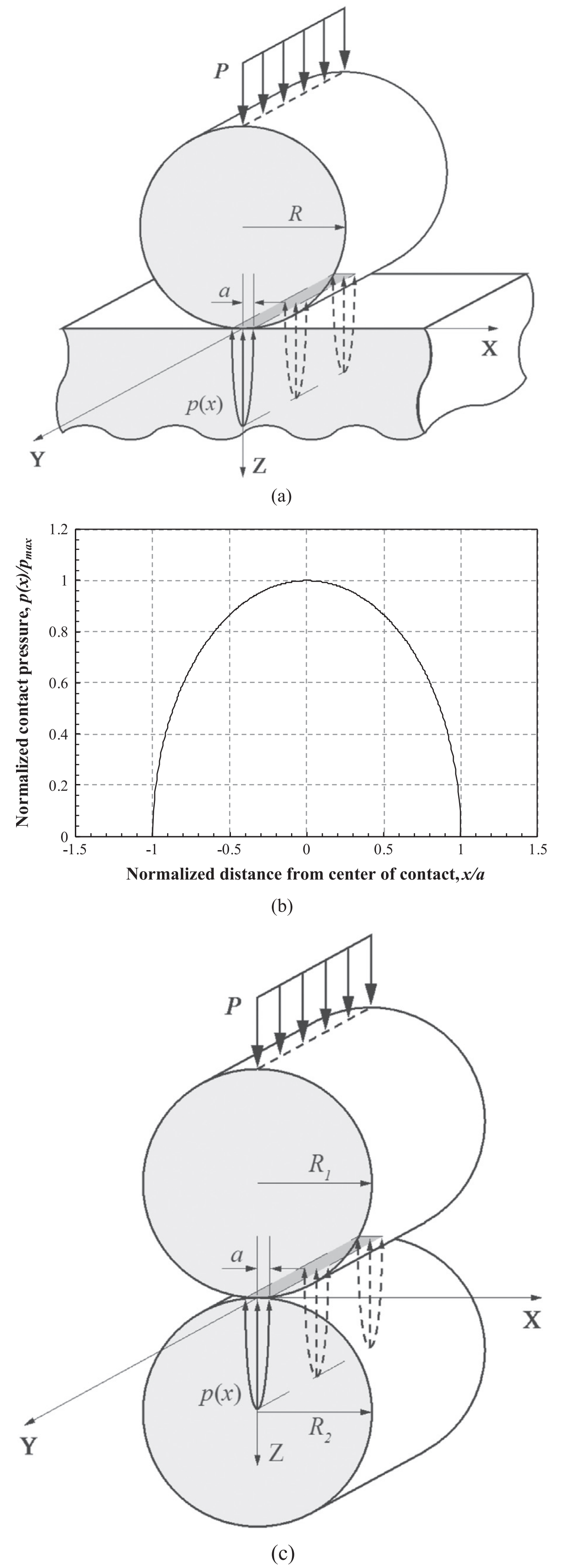

Fig. 1. (a) Contact between a cylinder and a flat surface due to a load per unit length. (b) The normalized elliptical contact pressure profile between a cylinder and a flat surface. (c) Contact between two cylinders due to a load per unit length.