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Statistical approaches to description of rough engineering surfaces at nano and microscales

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ABSTRACT

Statistical models of rough surfaces are widely used in tribology. These models include models based on assumption of normality of the asperity heights or similar assumptions that involve Gaussian distributions, models based solely on properties of the power spectral density of the surface heights along with models based on assumption of fractal character of roughness. It is argued that models describing surface roughness solely by its fractal dimension or its auto-correlation function (or its power spectral density) do not reflect tribological properties of surfaces. Then typical experimental data obtained for rough engineering surfaces prepared by grinding have been studied at nano and microscales. The heights of the micro-asperities were determined by a profilometre (stylus), while the data for nano/atomic scale was obtained by AFM (Atomic Force Microscopy). The assumption of the normal distribution for the roughness heights has been studied by application of various modern tests of normality. It was found that the height distribution of the surfaces under investigation were not Gaussian at both nano and microscales. Hence, the statistical models of rough surfaces under consideration cannot be used for description of the surfaces and there is a need in critical re-examination of the current statistical approaches.

1. Introduction

It is well established that topography of solid surfaces involves finite scale roughness regardless of preparation method of the surfaces [57,28,25]. In precision engineering practice surface roughness is a key factor in determining successful component performance, particularly in tribological applications (e.g. gears, bearings). Friction, wear and energy dissipation during sliding are strongly influenced by asperity deformation which is, in turn, controlled by the surface profile. Modern nanotechnology considers surfaces whose roughness is below the micro-metre scale and traditional statistical approaches to surface roughness have to be improved to achieve further progress in studying interactions at the nano/micro scales.

Often an analysis of rough surfaces starts from Fourier decomposition of the surface shape using sine and cosine functions. These functions create one of complete orthogonal sets of functions. If a surface profile is decomposed using non-trigonometric orthogonal sets of functions then this decomposition is called wavelet transform. A surface created by the Fourier synthesis may be described as

$$z(x) = \sum_{n=1}^{n_{max}} a_n \sin\left(\frac{2\pi n x}{2L} + \psi_n\right) \tag{1}$$

where ψ_n are random phases, and n_{max} is the number of used waves. Typically $a_n = a_1/n$ for engineering surfaces (see, e.g. [26,39]). The nominal shape of the surface is described normally by the longest spatial wavelength. Then the short wavelength shapes are referred to as “roughness”, and the long wavelength shapes represent “waviness” of the surface. One of drawbacks of the Fourier synthesis is that the region of definition of sine and cosine functions is infinite. Hence, the obtained surface is periodic, while the real surfaces are bounded domain where they are defined. On the other hand, wavelets are not equal to zero only within bounded regions. Hence, the boundary will affect only the narrow subregions near the boundaries of surfaces synthesized using the wavelet transform [14].

Modern contact solvers allow the researchers to simulate tribological problems with prescribed surface topography. Currently the topography may be described up to atomic scale resolution. However, the results of the simulations would be of little use because one needs to understand which parameters are governing for the tribological process under consideration and how the process behaviour will change when the parameters are varied

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[34,14]. Nowadays many parameters are used for characterization of the roughness [42]. As Whitehouse [54] noted some of these parameters are useful, but most are not. This confirms that there is no clear understanding what characteristics of rough surfaces are governing ones for tribological processes. Thus, one needs to understand the governed statistical characteristics of surfaces that determine the tribological behaviour at different scales of the real roughness. The present paper is one step in this direction. Various approaches for statistical analysis and representations of rough surfaces are discussed. The studies applied modern procedures for statistical analysis of roughness at nano and microscales. The tests of normality have been applied to experimental data obtained by the state-of-the-art experimental methods: atomic force microscopy (AFM) measurements of subnano/nano roughness and stylus measurements of microscale roughness of metallic surfaces.

The list of surface characteristics used includes various parameters and functions related to the asperity shapes and distributions of the asperities [42], e.g. height parameters, such as the maximum height of the profile; the root mean square (rms) parameters, such as the rms slope or the rms curvature; parameters associated with horizontal distributions, e.g. the number of intersections of the surface with the average line; parameters describing spatial extend of asperities, such as the high spot count, and so on. The maximum height of the profile R_{max}, the arithmetical mean deviation of the surface R_a, and the rms height R_q are the main statistical parameters for a function $z(x)$ describing the rough profile within an interval $[-L, L]$:

$$R_a = \frac{1}{2L}\int_{-L}^{L} |z(x)|\, dx, \quad R_{max} = \max_{x\in[-L,L]} z(x),$$
$$R_q = \left[\frac{1}{2L}\int_{-L}^{L} [z(x)]^2\, dx\right]^{1/2}.$$

Abbott and Firestone [1] suggested to calculate the integrals of the horizontal line at a specific level, which lies within the roughness profile, i.e. they introduced in tribology another important parameter of roughness (in fact, it is related to the cumulative distribution function of the surface heights). The parameter is called the bearing area curve or the Abbott–Firestone curve. Its value at a level z is equal to the length (the area in 2D problem) of the slice of the profile at the level z [1,26].

In addition to the above parameters, it was suggested to consider the surface roughness as a random process or signal (see, e.g. [31–33,57,40,41,28,26]). However, these approaches considered mainly microscale roughness and did not split the nano and microlevels, while it is important to realize that at angstrom, nano and micrometre scales different interaction mechanisms are involved in tribological processes.

Linnik and Khusu [31] and Whitehouse and Archard [57] suggested to study rough surfaces employing the profile auto-correlation function $R(\delta)$. Whitehouse and Archard [57] argued that a profile of a random rough surface can be represented by the waveform of a random signal that is completely defined by two parameters: a height distribution and an auto-correlation function

$$R(\delta) = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} [z(x+\delta)-\bar{z}][z(x)-\bar{z}]\, dx$$
$$= \langle [z(x+\delta)-\bar{z}][z(x)-\bar{z}]\rangle, \quad (2)$$

The latter parameter can be substituted by its Fourier transform – the power spectral density $G(\omega)$:

$$G(\omega) = \frac{2}{\pi}\int_0^{\infty} R(\delta)\cos\omega\delta\, d\delta \quad \text{and} \quad \bar{z} = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} z(x)\, dx \quad (3)$$

where $\bar{z}$ is the average value (the mean line) of the profile function $z(x)$, ω is the signal frequency and δ is the lag. The moments m_n of the spectral density $G(\omega)$ provides additional parameters of the surface roughness

$$m_n = \int_{\omega_0}^{\infty} \omega^n G(\omega)\, d\omega.$$

Here the profile length λ_0 correspond to the wavenumber $\omega_0 = 2\pi/\lambda_0$. For studies of rough surfaces, the structure function $S(\tau)$ given by

$$S(\tau) = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} [z(x+\tau)-z(x)]^2\, dx$$

is also employed in addition to the above described parameters and functions [55].

Employing the random signal approach, Sayles and Thomas [49] calculated the spectral density functions of many natural surface profiles and presented experimental relations between normalized G and wavelength. They showed that can be approximately presented as $G(\omega) = 2\pi\Lambda/\omega^2$ or

$$G(\omega) \sim 1/\omega^{\psi} \quad (4)$$

where the exponent $\psi = 2$ and Λ is called the topothesy of the surface, It follows from discussions by Whitehouse [56] and Borodich [12] that the latter term is not well defined. These results presented in logarithmic coordinates span nearly eight decades in wavelength. The fractal approach to surface roughness was trigged by Berry and Hannay [5] who argued that the surfaces are fractal and their geometric properties were discussed in detail by Mandelbrot [37]. For combining measurements of roughness for 23 types of various surfaces, the $\lg(G) \sim \lg(\omega)$ relation was presented as a united line spanned nearly eight decades in wavelength. This graph was the cause of many claims that surface topography has fractal behaviour through many orders of magnitude. Then the fractal approach was promoted by Berry and Lewis [6] by detailed discussion of the Weierstrass–Mandelbrot function and by providing a continuous approximation of its power spectrum.

The paper is organized as follows:

In Section 2 we give some preliminary information concerning various approaches to statistical studies of rough surfaces. In Section 2.1 we discuss the early approaches based on assumption of normality of the height distribution.

In 2.2 the fractal approaches and other models based on the use of power spectral density functions are critically examined. Although Borodich [7] and Borodich and Onishchenko [19] (see also [20,13]) gave several counterexamples showing that the fractal dimension alone cannot reflect the contact properties of the surfaces (in addition, [56]) gave a very negative appraisal to the use of fractals in tribology), papers devoted to the fractal approach to roughness prefer to avoid any discussion of these arguments. Hence, the fractal approach is discussed in this paper and it is argued many approaches that pretend to be the fractal ones, are based not on scientifically justified information but on myths about fractals and the use of ill-defined terminology, i.e. instead of the state-of-the-art approaches they use the state-of-the-street ones.

In Section 2.3 it will be shown that the approaches that uses solely the surface power spectrum $G(\omega)$ for representation of the surface roughness have in fact the same drawbacks as the above fractal ones. It is argued that the surface power spectrum alone cannot reflect the contact properties of the surfaces.

One of the purposes of the paper is testing of normality of the height distribution of the roughness. This is because the classic statistical approaches [31–33,57,40,41,28] assume normality of the height distribution of the rough surfaces. It is known that *normality (or lack thereof) of an underlying data distribution can have an effect to a greater or lesser degree on the properties of estimation or inferential procedures used in the analysis of the data* [50]. Thus, in Section 2.4 seven established tests of normality of data are