Tribology International 102 (2016) 182–197

Contents lists available at ScienceDirect

Tribology International

journal homepage: www.elsevier.com/locate/triboint

# An extended Reynold equation applicable to high reduced Reynolds number operation of journal bearings

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ARTICLE INFO

*Article history:*
Received 22 December 2015
Received in revised form
27 March 2016
Accepted 30 April 2016
Available online 16 May 2016

*Keywords:*
Temporal inertia effects
Convective inertia effects, and turbulence
Journal bearings
Extended Reynolds equation

ABSTRACT

The introduction of this paper serves as a review of previous studies on extending the Reynolds equation. This paper develops an extended Reynolds equation and includes turbulence and inertia effects. Use of low viscosity lubricants and/or high rotational speed applications yield high Reynolds and high reduced Reynolds numbers. Our approach fully realizes the convective inertia effects on the static and dynamic properties of journal bearings as nonlinear with reduced Reynolds number. Two types of temporal inertia terms are identified for the first time: primary and secondary, in both laminar and turbulent regimes. The contribution of the secondary inertia effects can be up to 30% of the lubricant added mass coefficients. Lubricant added mass coefficients are potentially comparable to journal mass.



## 1. Introduction

Low viscosity lubricants are commonly used in applications such as water bearings in pumps, including nuclear water pumps, refrigeration compressor bearings (where the oil is often mixed with R134a and other refrigerants), process lubricated bearings in canned pumps, and potential supercritical $CO_2$ lubricated bearings in closed Brayton power cycles [1]. The hydrostatic bearings of the high pressure turbopumps used in space shuttle main engines are lubricated with low viscosity liquid hydrogen and oxygen. These turbopumps operate at very high speeds. To minimize the parasitic losses and system's rotordynamic simplicity, it is desired to remove oil lubricated gear trains from HVAC applications and move to direct drive systems. In this concept, the oil based lubricant leakage is a challenge, which makes its replacement with the refrigerant alluring to manufacturers. In the low viscosity lubricant bearings, the inertia effects are important and the reduced Reynolds number is typically larger than one, in contrast to most oil lubricated bearings. Usually inertia effects are neglected and the reduced Reynolds numbers are much less than one in oil bearings. The static and dynamic properties of these low viscosity lubricated bearings are significantly affected. Another application is that of squeeze film dampers where a reduced Reynolds number may be very larger than unity.

Once inertia effects are included, the stiffness and damping coefficients employed in rotordynamic calculations will have different numerical values and added mass coefficients will appear. The main concern in these bearings is the low viscosity of the alternative lubricants compared to oil, which increases $\mathbf{Re} = \frac{\rho U c}{\mu}$ and $\mathbf{Re}^* = \mathbf{Re}\frac{c}{R}$ drastically and makes these bearings prone to significant turbulence and fluid inertia effects. The linear approximation of the fluid film hydrodynamic forces via the stiffness, $K$, and damping, $C$, coefficients as offered by traditional Reynolds equation, is not adequate and must include the lubricant added mass, $M$, i.e., $F = -KX - C\dot{X} - M\ddot{X}$. In addition, the stiffness and damping coefficients should be calculated with the convective inertia and turbulence effects taken into account. We also note that oil bearings in some applications, such as high speed compressors, may raise the issue of the inertia effects on the dynamic properties as well.

Many researchers have shown an interest in investigating the possibility of further improving the classical Reynolds equation to include turbulence and inertia effects. Turbulence occurs in applications with high Reynolds numbers. In large **Re** applications, turbulence effects appear, as indicated by Wilcock [2], in the form of increased power consumption, reduced oil flow and a sharp change in bearing eccentricity. These phenomena are attributed to the increase of the *effective* viscosity due to the random mixing of the lubricant. In an effort to develop a turbulent lubrication model consistent with channel flow data, Ng [3] postulated the application of Reichardt's eddy diffusivity formulation [4] to a shear stress formulation in the turbulent regime. Ng and Pan [5] extended this theory based on a linear flow-pressure model. Elrod and Ng [6], and Safar and Szeri [7] also developed turbulent Reynolds equations with the eddy viscosity assumption, but their flow-pressure

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http://dx.doi.org/10.1016/j.triboint.2016.04.037
0301-679X/

**Nomenclature**

| | |
|---|---|
| $\mathbf{Re}^*$ | $=\mathbf{Re}\frac{c}{R}$, reduced Reynolds number |
| $\mathbf{Re}$ | $=\frac{\rho U c}{\mu}$, Reynolds number |
| $\mu$ | lubricant viscosity |
| $\rho$ | lubricant density |
| X,Y | horizontal and vertical directions, coordinates and displacements of journal Fig. 1 |
| $\alpha,\beta,\gamma,\delta$ | coefficients of Constantinescu's formula for inertia parameters |
| $\overline{A}$ | nondimensional form of $A$ |
| $\ddot{A}$ | $=\frac{\partial^2 A}{\partial t^2}$ |
| $\dot{A}$ | $=\frac{\partial A}{\partial t}$ |
| $\epsilon$ | $=e/C$ radial eccentricity ratio |
| $\Gamma$ | convective inertia correction term in Constantinescu's formulation |
| $\hat{i},\hat{j}$ | Cartesian coordinates, horizontal and vertical |
| $\hat{n}_r,\hat{n}_\theta$ | polar coordinates, radial and circumferential directions |
| $\omega$ | journal rotational speed (rad/s) |
| $\phi$ | attitude angle, angle of line of journal and bearing centers with $x$ coordinate |
| $\sigma$ | pressure solution error at each iteration |
| $\tau$ | viscous shear stress |
| $\tau_w$ | wall viscous shear stress in Hirs' formula |
| $\theta$ | circumferential direction and variable |
| $\zeta,\chi$ | $=X,Y$ coordinate dummy variables |
| $A_{,a}$ | $=\frac{\partial A}{\partial a}$ |
| $A_o$ | steady state value of variable $A$ |
| $C$ | damping coefficient/matrix |
| $c$ | radial clearance |
| $D$ | bearing diameter |
| $F$ | hydrodynamic force |
| $F_r,F_\theta$ | radial and tangential forces ($N$) |
| $h$ | film thickness |
| $h_o$ | steady state film thickness |
| $i$ | node number of the FEA grids |
| $I_c$ | convective inertia |
| $I_{ij}$ | direct or cross coupled convective inertia parameter |
| $I_t$ | temporal inertia |
| $K$ | stiffness coefficient/matrix |
| $k_x$,$k_z$ | Constantinescu's turbulence coefficients |
| $L$ | bearing length |
| $M$ | lubricant added mass coefficient/matrix |
| $m_o$,$n_o$ | experimentally obtained constants of Hirs' wall shear formula |
| $N$ | journal rotational speed (rpm) |
| $O_s$,$O_b$ | bearing and shaft centers |
| $p$ | hydrodynamic pressure |
| $q$ | unit length flow rate, [$L^2\ T^{-1}$] |
| $Q_x,Q_z$ | distribution coefficients of the time derivative of flow rate gradients in $x$ and $z$ directions |
| $R$ | journal radius |
| $S$ | $=\left(\frac{R}{c}\right)^2\frac{2\mu NLR}{W}$, Sommerfeld number |
| $t$ | time |
| $U$ | surface velocity of journal |
| $u$ | local fluid velocity along direction of rotation |
| $u_i$ | general fluid velocity in $i$ direction |
| $U_p,W_p$ | average velocity of the pressure driven Poiseuille flow in $x$ and $z$ directions |
| $v$ | local fluid velocity across the film thickness |
| $w$ | local fluid velocity along axial direction |
| $W$ | $=\sqrt{F_{Xo}^2+F_{Yo}^2}$, external force and load carrying capacity |
| $x$ | circumferential direction, horizontal direction of an unwrapped film, $x=R\theta$ |
| $x_L$,$x_T$ | leading and trailing edges location |
| $x_{cav}$ | downstream cavitation onset location |
| $x_{pmax}$ | maximum pressure location in the circumferential direction |
| $y$ | vertical direction equivalent to negative radial direction of an unwrapped film |
| $z$ | axial direction |

relationship was nonlinear. This method, in spite of requiring an iterative scheme, is widely used in the literature for different bearings and seal applications [8–11]. An alternative methodology of including turbulence effects was suggested by Hirs [12]. This method does not attempt to analyze turbulence in detail, and instead, it is based on the existing measured global characteristics of flow, namely the relationship between wall shear stress and the average velocity

$$\frac{\tau_w}{\frac{1}{2}\rho U_a^2}=n_o(\mathbf{Re})^{m_o}. \tag{1}$$

where $U_a$ is the average film velocity and $\mathbf{Re}=\frac{U_a h}{\nu}$ is the Reynolds number based on $U_a$. In thin film lubrication theory, there is no general agreement of the exact transition region or onset of turbulence. For instance, Taniguchi et al. used $1000<\mathbf{Re}<1500$ as transition region limits. However, Xu and Zhu demonstrated in their experiments that the values of the critical Reynolds numbers are functions of the eccentricity [13,14]. This study adopts 500 and 1000 for the aforementioned limits.

Most of the aforementioned studies confine the high Reynolds number phenomena in modeling turbulence only and with no inclusion of the inertia effects. On the other hand, some studies have focused on the inertia effects in the laminar regime. Mulcahy [15] and Brennen [16] were among the early researchers who explored the effects of fluid inertia on whirling shafts. An early study on fluid inertia contributions to laminar plain journal bearings was conducted by Reinhart and Lund [17]. They assumed a small perturbation expansion of unknowns of Navier–Stokes equations using a Taylor's series expansion in reduced Reynolds number which led to zeroth and first order governing equations. The important short coming of this method was its divergence for large $\mathbf{Re}^*$, as mentioned by Grimm [18]. Reinhart and Lund concluded that the added mass coefficient can be significant in applications with short rotors. He et al. [19] adopted this method and extended it to tilting pad bearings and showed that in water bearings inertia effects are important although their results also carry the limitations on the assumption of $\mathbf{Re}^*\ll 1$ associated with the method by Reinhart and Lund. In more practical terms, Tichy quantified the importance of the fluid inertia forces in squeeze film flows with the analysis of viscoelastic fluids for small centered and off-centered journal whirling motions [20–22].

In general, the convective inertia terms in Navier–Stokes equations are nonlinear products of the velocity components, which result in integrals of the form

$$I_{ij}=\int_0^h u_i u_j\,dy,\quad i,j=x,z \tag{2}$$

where $I_{ij}$ are inertia parameters. The early attempts [12,23–27] to