Tribology International 97 (2016) 71–76

Contents lists available at ScienceDirect

Tribology International

journal homepage: www.elsevier.com/locate/triboint

Hertzian load–displacement relation holds for spherical indentation on soft elastic solids undergoing large deformations

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ARTICLE INFO

Article history:
Received 3 November 2015
Received in revised form
20 December 2015
Accepted 23 December 2015
Available online 12 January 2016

Keywords:
Hertz contact theory
Nanoindentation
Soft materials

ABSTRACT

In this paper we investigate the validity of the Hertz theory at very large deformations by performing rigid spherical indentation on soft, linearly elastic silicone substrates using nanoindentation tests and finite element method. We show that the theory significantly underestimates and overestimates the contact radius and maximum contact pressure, respectively, as the ratio of indenter displacement δ to the indenter radius R exceeds 0.1. However, the loading load–displacement relation still holds ($<3\%$ discrepancy) for δ/R as large as 0.66 with a maximum principal strain of 46.6%. This agreement arises from a near cancellation of two non-Hertzian effects: the spherical (as opposed to paraboloidal) shape of the indenter, and the large deformation behavior of the linear elastic system. Our simulation results show that the Hertzian load–displacement relation does not hold for thin films where the ratio of thickness H to R is smaller than 20. We also consider rigid indentation on an elastic sphere with a radius of kR, and reveal that the elastic sphere is large enough to be treated as a half-space for $k>10$. Our results may provide practical guidelines to proper sample preparation and better interpretation of indentation data of elastic soft elastomers and gels.

1. Introduction

Contacts between deformable solids are ubiquitous in nature and engineering, and have important roles in various fields and applications from physics [1,2], biology [3,4], agriculture [5] and astrophysics [6] to nanoindentation [7,8], powders [9], magnetic disk drives [10] and railways [11]. The Hertz theory has been the cornerstone of modern contact mechanics since Hertz published his classic paper in 1882 [12]. It describes the normal contact between two perfectly elastic solids, and has been successful in predicting the load–displacement relation as well as other parameters such as contact radius and contact pressure. The theory uses parabolic approximation for the profile of the sphere, which is only valid for small contact radii. As a result, one of the major assumptions of the theory is small deformation, i.e. the contact area is generally small compared with the contacting bodies themselves [12–17].

The nanoindentation technique has become an important means to measure mechanical properties of soft and biological materials including cells [18], soft elastomers and gels [19]. Whereas the Hertz theory was originally developed to describe macroscopic contacts with infinitesimal strain, it has been widely applied to interpret the indentation data of soft materials where the deformation is often not small or the sample thickness is in the nanometer to micrometer regime [19]. It has been shown that shallow depth indentation often suffers from excessive noise which may preclude the accuracy of the measurement [7]. Relative larger indentation depth can help reduce the measurement noise due to surface effects, but the interpretation of the data become more challenging.

There is a wealth of literature on the applicability of the Hertz theory to the interpretation of indentation data. Many have focused on elastic–plastic problems of metallic solids [7,20], which show a reversible linear elastic behavior only to 1% of the strain. Dintwa et al. [21] investigated the validity of the Hertz theory, for two contacting elastic spheres and contact of an elastic sphere on a rigid flat, using finite element method (FEM), and concluded that large strains cause important prediction errors in the Hertz theory – for both cases the Hertz model systematically underestimates the normal force even at a relative small indentation. Yoffe [22] published pioneering theoretical work on the modification of Hertz theory for spherical indentation for wider contact. Lim et al. [23] carried out pioneering experimental work on PDMS and other rubbers using 4-mm diameter spherical indenter. They confirmed that the experimental loading curves were well fitted by the Hertz equation with a δ/R up to 0.15, where R is the indenter radius and δ the indenter displacement. A large amount of work has been carried out on the indentation of soft solids considering material

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http://dx.doi.org/10.1016/j.triboint.2015.12.034
0301-679X/

nonlinearity such as hyperelasticity [13,15,24–27], viscoelasticity [28,29] and poroelasticity [30]. Most of the work focused on the identification of the nonlinear material properties or the constitutive parameters from fitting measured load–displacement data using inverse methods. Although these nonlinear models are comprehensive and provide insight into the nature of soft materials, the inverse method often requires a numerical optimization procedure [26] in contrast to the Hertzian theory by which the Young's modulus can be readily obtained. Accordingly, the Hertzian load–displacement curve is still widely used by researchers to extract the Young's modulus [19] without knowing the level of accuracy. Recently Nalam et al. [31] measured local material properties of hydrogels using colloid-attached atomic force microscope probes ($R=2.5\ \mu m$) in liquid with a maximum indentation depth of 350 nm. During the loading process at low frequencies, their measured force-indentation relation agreed with the Hertz model, and the load was not affected by interfacial bonding during approach. Despite the extensive work, the applicability of the Hertzian theory for very large indentation ($\delta/R=1$) remains unclear, and a better quantitative understanding and a direct comparison between the theory, experiment, and numerical simulation are still lacking.

Here we quantify the extent of deviation from the Hertzian theory which result from geometrical nonlinearities in the large strain regime by performing rigid spherical indentation on soft, linearly elastic silicone substrates using nanoindentation tests and finite element method. Material properties are directly measured by tensile tests, instead of by fitting the indentation data, to avoid any additional uncertainty.

Fig. 1 shows a schematic of the contact between a rigid spherical indenter and a deformed elastic half-space as a result of a normal load F. The half-space is assumed perfectly elastic, homogeneous and isotropic. Hertz approximated the original separation between the axisymmetric contacting bodies by $z=(x^2+y^2)/2R=r^2/2R$, where r is the radial coordinate, z the axis of symmetry and x–y the common tangent plane of the two bodies. Hertz also proposed the contact pressure distribution p of the circular contact as $p=p_0\ [1-(r/a)^2]^{1/2}$, where a is the contact radius, p_0 the maximum contact pressure on the surface and it occurs at the center. The maximum pressure is given by $p_0=3F/2\pi a^2$, where F is the normal load. The well-known Hertzian relationship between the load F and displacement δ is given by

$$F=\frac{4}{3}E^{*}R^{0.5}\delta^{1.5} \tag{1}$$

where $E^{*}=E/(1-\nu^2)$ is the equivalent elastic modulus, E and ν are the Young's modulus and Poisson's ratio of the elastic half-space, respectively.

2. Experimental

Some silicone gels and elastomers [32–34] are known to exhibit linearly elastic behavior at a strain larger than 20%, and are suitable as the test material for this work. We prepared polydimethylsiloxane (PDMS, Sylgard 184, Dow Corning), a silicone elastomer, following an existing protocol [34]. It was prepared by mixing an elastomer base and a cross-linker at a weight ratio of 10:1 for 10 min with a spatula, followed by degassing. The prepolymerized mixture was then poured into a mold, degassed, and then cured at 80 °C for 24 h. After the hardening process, the sample was taken out of the mold and glued to two acrylic clamps, resulting in specimens measuring $60.0\times 20.0\times 3.0\ mm^3$ and the rectangular test section measuring $25\times 20.0\ mm^2$. All tensile tests were performed in air, at room temperature, using a tensile machine (Criterion 42.503 Test System, MTS) in the displacement-controlled mode at a constant rate of 0.5 mm/min. We use camera (D90, Nikon) with micro lens (AF-S MICRO NIKKOR 105 mm 1:2.8 G ED, Nikon) to take images of the test section at the center of the sample during the test (Fig.2).

The stress–strain curve is obtained using the test section, and hence the effect of the instant glue and acrylic clamps can be excluded. We confirmed that test section can be regarded as under uniaxial tension by finite element simulations. Fig. 2(c) shows the stress–strain curves and the Poisson's ratio as a function of the tensile strain ε_y. The Poisson's ratio, $\nu=0.412\pm 0.00924$, was calculated as the ratio of transverse strain ε_x and tensile strain ε_y, which were estimated from the images of the test section. For $\varepsilon_y<0.05$ the transverse strain ε_x was too small to resolve reliably using our experimental setup so the Poisson's ratio was not

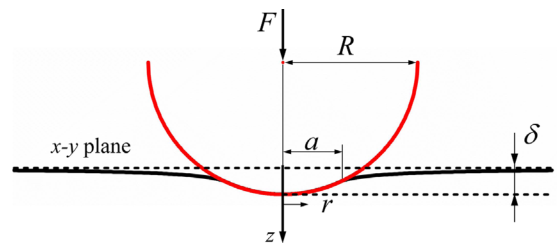

Fig. 1. Schematic diagram of the spherical indentation.

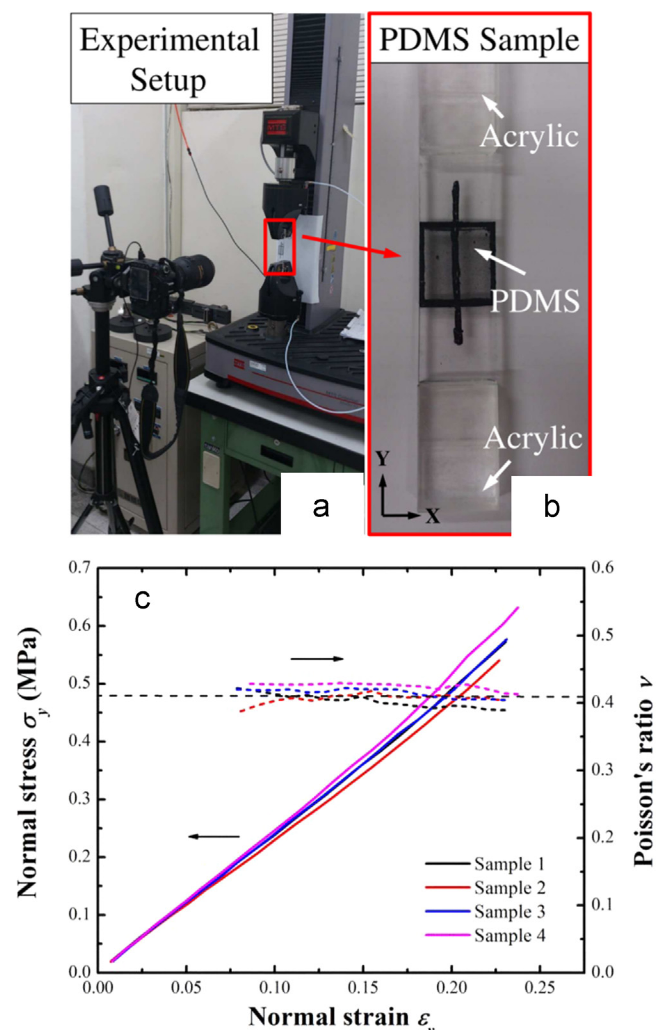

Fig. 2. (a) Experimental setup and (b) PDMS sample for our tensile test. (c) The measured stress-strain curves and Poisson's ratios. The stress is calculated by P/A, where P is the applied load, and A is the measured cross-sectional area of the deformed test section. The Young's modulus and Poisson's ratio are 2.463 ± 0.117 MPa and 0.412 ± 0.00924, respectively.