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Transversely isotropic viscoelastic materials: Contact mechanics and friction

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ABSTRACT

Transversely isotropic materials are a unique group of materials whose properties are the same along two of the three principal axes. Various natural and artificial materials behave effectively as transversely isotropic elastic solids therefore this specific case of anisotropy has several engineering and industrial applications. Various components can be classified as transversely isotropic materials including crystals, rocks, piezoelectrics, biological tissues such as muscles, skin, cartilage tissue or brainstem and fibrous composites. In this study, the theory of contact mechanics developed by Persson is extended in such a way that it can model the contact and friction of a transversely isotropic viscoelastic solid in contact with a rigid rough surface. Numerical results show that anisotropy should be taken into account when dealing with transversely isotropic solids. The experimental results validate the theory.

1. Introduction

The interaction between two contacting solids which is the main subject of tribology plays a major role in a large number of physical phenomena and engineering applications such as friction, lubrication and wear. The analysis of the stresses, contact stiffness, surface deformations and contact areas generated by the contact between bodies with rough surfaces needs to be accurately accounted for in a smart design of various engineering components including but not limited to tires, seals, bio-inspired adhesives, coatings, piezoelectric materials, electrical contacts and syringes. The mechanics of isotropic elastic materials is a quite developed research field. The contact problems for anisotropic solids were studied in many papers [1–4]. Transversely isotropic materials are a unique group of materials whose properties are the same along two of the principal axes. Several materials can be classified as transversely isotropic materials including crystals, rocks, piezoelectric materials, some biological tissues such as muscles, skin, cartilage tissue or brainstem and fibrous composites. Researchers have been investigating the mechanics of transversely isotropic materials for many years [1,3,5–8] simply because many natural and artificial materials behave effectively as transversely isotropic elastic solids. Viscoelasticity which makes the contact problem even more complex, is crucial in modelling rubber-like materials and biological tissues and should not be ignored.

Several approaches dealing with the contact of real random rough surfaces have been introduced. Greenwood and Williamson (GW) proposed asperity-based models [9], in which the roughness is reduced to a set of identical asperities distributed according to a Gaussian or exponential height distribution. The GW model has attracted several researchers for a long time. Every asperity is incorporated with a Hertzian punch in this model. Little is gained by proposed modified versions of the GW contact model, treating the asperities as ellipsoids, or by introducing a distribution of asperity sizes [10]. Random process theory is used in other modified versions of the GW model to make the asperity curvature depending on their heights. In another attempt, fractal theory is used to recognize the multiscale nature of real surfaces [11]. Although the achieved results through multi-asperity contact models are of practical interest, neglecting the interactions between neighboring microcontacts is the main disadvantage of such models. In the study of soft materials like rubbers or soft biological tissues, the effect of the asperities on each other cannot be neglected because these flexible materials can deform much easier and therefore the problem of neglecting interactions between neighboring asperities is heavier especially when approaching full contact, i.e. when the contact spot separation is of comparable size with the spot size itself. The interaction between the asperities can be added to the current asperity models [12,13], however, these approaches remain quite approximate [14]. On the other hand, a new model was proposed by Persson [15] that does not pre-exclude any scale of roughness from the analysis. In his

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model, the exact solution for the case of full contact is firstly obtained and further analysis is extended to the partial contact by imposing a boundary condition which is an approximate solution. Even though Manners and Greenwood [16] raise some concerns about the boundary conditions applied in Persson's theory, the behavior of elastically soft materials (with the ability to bend and fill out the roughness on at least small wave lengths) is more analogous to Persson's analysis, than the asperity contact models (where it is assumed that contact occurs on segregated islands, far from each other, which are named asperities and do not have any influence on each other because of the far distances in between). Persson introduced a correction of the expression used for the elastic energy in the asperity contact regions [17]. A comparison between numerical results and Persson's contact model has led to a suitable value for the correction factor [18]. Multi-asperity contact models and Persson's approach have been comparatively analyzed [19].

The theory of contact mechanics and friction of Persson has been extended to model the contact and friction between surfaces with anisotropic surface roughness [20]. The main purpose of this study is to extend Persson's theory in such a way that it can model the contact and friction of a transversely isotropic viscoelastic solid in contact with a rigid rough surface. This is of importance in modelling the contact of soft biological tissues such as, muscles, skin and brainstem. Mechanical properties of the novel composites can be controlled and tuned by morphology, distribution and alignment processes to achieve the desired characteristics [21]. Moreover, fiber reinforced composites can (depending on the direction of the fibers in the composite) show transversely isotropic characteristics. They have a wide range of industrial applications such as tires, transmission belts and seals. In Section 2, the theory is explained. In the next section, both the real area of contact and the viscoelastic contribution to the overall friction of a transversely isotropic material are calculated and the results are compared with the results for an isotropic material. The measured dynamic mechanical properties (in different directions) of a prepared unidirectional fiber reinforced rubber sample as well as the measured friction between two rough granite surfaces and the rubber sample are presented in Section 4. The experimental results are compared with the numerical results of the theory in Section 5 and the conclusions are summarized in Section 6.

2. Friction between a transversely isotropic viscoelastic material and a rough rigid surface

The friction contributors in the contact between a viscoelastic solid and a rigid rough surface are commonly described by two main contributors i.e. the adhesion component and the hysteresis component [22]. Hysteresis component of friction is generated by (cyclic) deformation of the rubber which dissipates energy via the internal damping in the bulk of the material [15]. Adhesion is characterized by the attractive forces between the contacting bodies [23]. Energy dissipation due to crack opening [24] and energy dissipation in shearing of a thin viscous film [25] are other contributors to friction of elastomers. Pan [26] has shown the significant role of interfacial interactions, in addition to the bulk viscoelastic hysteresis, in determining the wet sliding friction of elastomer compounds. The friction force contributions mentioned before are summarized in terms of two main forces (see Fig. 1): (1) the contributions related to the viscoelastic deformations; and (2) the contributions related to the real area of contact as defined in Eq. (1). One contributor cannot be indicated as the main contributor to the friction as a generalized rule [27], but depending on the tribological conditions, hysteresis or the real area of contact can play a dominant role in determining the overall friction.

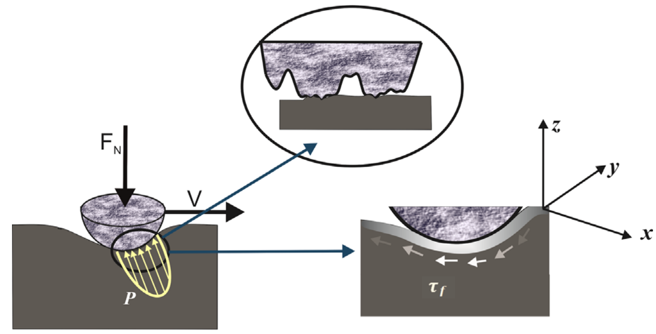

Fig. 1. A granite semi–sphere in sliding contact with a transversely isotropic viscoelastic half space is shown schematically. The contributions to friction from (i) shearing of a thin modified surface layer (τ_f) and (ii) hysteresis in the bulk of the rubber are demonstrated.

$$F_f = F_{visc} + \tau_f A_{real} \tag{1}$$

where F_f, F_{visc} are the forces concerning the total friction and the contribution from the hysteresis losses respectively and the product $\tau_f A_{real}$ represents the force in the real area of contact where τ_f, A_{real} are the frictional shear stress and real area of contact. In some tribological systems, the shear stress between the two contacting bodies changes according to the tribological conditions and the frictional energy input [28].

2.1. Contact mechanics between a transversely isotropic viscoelastic material and a rough rigid surface

Consider a rough rigid surface sliding at a constant velocity $\boldsymbol{v}$ on a transversely isotropic viscoelastic half space (whose surface is parallel to the planes of isotropy). Take a rectangular coordinate system $(\mathbf{x}, z) = (x, y, z)$. By the application of a concentrated load $F(\mathbf{x}, 0) = F_0$ on the free surface of the transversely isotropic viscoelastic solid, the displacement at any point on the surface, $u_z(\mathbf{x}, 0)$, can be calculated by the equation below [29], substituting $z = 0$:

$$u_z(\mathbf{x}, z) = -\sum_{i=1,2} \frac{\alpha - \gamma s_i^2}{s_i^2} \frac{\partial \varphi_i}{\partial z} \tag{2}$$

In the above equation,

$$\begin{cases} \varphi_1(\mathbf{x}, z) = -\frac{s_1[\alpha + (1-\gamma)s_2^2]}{s_2 k_2[\alpha + (1-\gamma)s_1^2] - s_1 k_1[\alpha + (1-\gamma)s_2^2]} \left(\frac{F}{2\pi B_{66}}\right) \log\left(\sqrt{x^2 + y^2 + s_1^2 z^2} - s_1 z\right) \\ \varphi_2(\mathbf{x}, z) = -\frac{s_2[\alpha + (1-\gamma)s_1^2]}{s_2 k_2[\alpha + (1-\gamma)s_1^2] - s_1 k_1[\alpha + (1-\gamma)s_2^2]} \left(\frac{F}{2\pi B_{66}}\right) \log\left(\sqrt{x^2 + y^2 + s_2^2 z^2} - s_2 z\right) \end{cases}$$

and the constants $\alpha = \frac{B_{11}}{B_{13}}, \gamma = \frac{B_{44}}{B_{13}}, s_0^2 = \frac{B_{66}}{B_{44}}, k_i = \frac{1 - \gamma - \beta(\alpha - \gamma s_i^2)}{\gamma s_0^2}, \beta = \frac{B_{33}}{B_{13}}$ are related to the 5 elastic constants. In addition,

$$s_{1,2}{}^2 = \frac{B_{44}{}^2 + B_{11}B_{33} - B_{13}{}^2 \pm \sqrt{(B_{44}{}^2 + B_{11}B_{33} - B_{13}{}^2)^2 - 4B_{11}B_{33}B_{44}{}^2}}{2B_{33}B_{44}}.$$

If the generalized Hook's law for a transversely isotropic solid, with symmetry plane x–y is written:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{zx} \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & 0 & 0 & 0 \\ A_{12} & A_{11} & A_{13} & 0 & 0 & 0 \\ A_{13} & A_{13} & A_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & A_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & A_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & A_{66} \end{bmatrix} \begin{bmatrix} \frac{\partial u_x}{\partial x} \\ \frac{\partial u_y}{\partial y} \\ \frac{\partial u_z}{\partial z} \\ \frac{\partial u_z}{\partial y} + \frac{\partial u_y}{\partial z} \\ \frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z} \\ \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \end{bmatrix} \tag{3}$$

Then the following relations hold: