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Forced stick-slip oscillations allow the measurement of the friction force: Application to paper materials

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ABSTRACT

Different methods for measuring the friction forces are investigated in this paper. We consider the paper-on-paper contact as an example of application. We first underline several drawbacks for the two main standard methods, namely the inclined and horizontal plane methods. In particular, the horizontal plane test method often involves stick-slip oscillations that make the measurement of the friction force impossible. We then propose a method for characterizing these oscillations and removing their influence on the friction force measurement. The comparison of the proposed method to standards suggests that our proposed method delivers measurements that are much more accurate and repeatable. We finally discuss the validity of averaging the friction force measured during the sliding movement.

1. Introduction

The friction phenomena being complex and not totally mastered, their study remains mainly empirical. The paper-on-paper contact [1] illustrates the study, but the results can be generalized to other contacts. The standards in the context of ISO, TAPPI, or AFNOR are based on the inclined and horizontal plane methods [2–9]. However, these methods suffer from major issues.

The inclined plane method consists in fixing one sample on a plane and the other on a weighted sled. The plane is then tilted from the horizontal to a critical angle, α, at which the sled starts to slide, as represented in Fig. 1a.

As the sliding starts, the friction force is called *break-away force*. The coefficient of static friction, μ_S, is then defined as the dimensionless ratio between the break-away force and the normal load, and characterized by the critical angle. The method is cheap, intuitive, and easy to run. However, we underline several drawbacks hereafter.

(i) The coefficient of static friction sometime depends on the normal load, e.g., in the case of polymers [10,11]. However, different coefficients of static friction measured with the inclined plane correspond to different normal loads. A comparison between different coefficients of friction is therefore impossible or requires multiple measurements with varying loads.
(ii) The repeatability of the measurement is poor ($\pm 2\%$) due to the difficulty to observe the beginning of the sliding. As we observe in our experiments, it is in particular the case for the rubber-on-steel contact, due to a large difference of stiffness of the materials [12].
(iii) As the sliding is not controlled, this method cannot be used to study the effect of repeated tests on a single couple of samples. This is a major drawback if the coefficient of static friction evolves with repeated slidings, as for example in the case of paper-on-paper contacts [13].
(iv) The method is useful to study only simple models of friction. In particular, the method does not allow the characterization of the kinetic friction and presliding displacements.

On the other hand, the horizontal plane method consists in measuring the pulling force, F_p, required to move the sled at constant speed, as represented in Fig. 1b. The maximum pulling force is considered as the break-away force and recorded to calculate the coefficient of static friction. Moreover, when the sled reaches the chosen velocity, the pulling force is averaged and considered as the *force of kinetic friction*. The coefficient of kinetic friction, μ_K, is then defined as the dimensionless ratio between the force of kinetic friction and the normal load. This method maintains the normal load constant and is adapted to the study of repeated slidings. However, we underline several drawbacks hereafter.

(i) The maximum pulling force does not necessarily correspond to the beginning of the sliding. For example, viscous friction may induce small displacements as the force of friction gets

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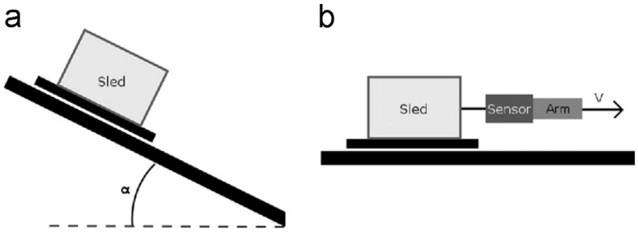
Nomenclature

x	position of the sled (m)
x_a	position of the arm (m)
l_0	spring elongation at rest (m)
u	spring elongation (m)
w	interpolation of the spring elongation (m)
k	stiffness of the sled–spring–sensor system (N m^{-1})
F_p	pulling force (N)
F_f	force of friction (N)
m	mass of the sample–sled system (kg)
g	standard gravity (m s^{-2})
F_N	normal load (N)
μ_S	coefficient of static friction (–)
μ_K	coefficient of kinetic friction (–)



Inclined plane

Horizontal plane

Fig. 1. The standard methods for friction force measurement. (a) Inclined plane. (b) Horizontal plane.

established. We exemplify this problem in the rubber-on-steel case in the supplemental materials.

(ii) The acceleration is often neglected in the calculation of the friction force. Moreover, the determination of the maximal pulling force is based on a low number of measured points that decreases with the acceleration. As a consequence, the errors due to the acceleration of the sled increase with the acceleration. In particular, this situation is critical when the initial acceleration of the sled is produced by a shock between the force sensor and the sled, as described by several standards [3–5].

(iii) The method does not allow the separation of the force components due to static friction, kinetic friction, and mass-acceleration [2,14]. Avoiding stick-slip by increasing the stiffness of the sensor and/or the velocity of the displacement [15] would increase the sled acceleration and therefore the errors described in (ii). We remind that the stick-slip consists in a sequential build-up and release of stored energy in elastic components, resulting in cyclical acceleration and deceleration of the sled.

(iv) The method does not allow the measurement of micro-displacements which may be required for dynamic models of friction [16].

We underlined several drawbacks of the standard methods used to measure the friction force. These drawbacks limit the characterization of friction. Defining a protocol improving both the reproducibility and the range is therefore a major issue we propose to explore. We built a new experimental setup to conduct various experiments. The results are compared to standard methods. Advantages of the proposed setup and possible future improvements are finally discussed.

2. Materials and methods

2.1. Proposed setup

We use an horizontal plane tribometer in accordance with the standards (NF Q 03-082 [5] and TAPPI 549 [3]). The sled weights 837 g and its dimensions are 60 mm × 60 mm. The velocity of the arm is set to 5 mm s^{-1}. The proposed setup, called *oscillating setup*, consists in placing a spring between the force sensor and the sled. The sled–spring–sensor system has a constant spring stiffness, k (390 N m^{-1}). The spring induces a stick-slip phenomenon at roughly 2 Hz. We plug an analog filter to decrease the noise delivered by the force sensor. A Linear Variable Differential Transformer (LVDT) position sensor (accuracy ±0.01 mm) is placed between the sled and the arm, parallel to the spring, as represented in Fig. 2.

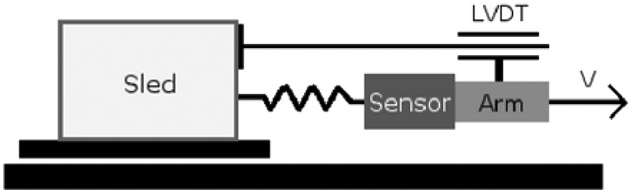


Fig. 2. Schematic view of the developed setup. A spring is placed between the sled and the force sensor. An LVDT measures the spring elongation.

The frequency of acquisition is 400 Hz. The measurements are processed using a Labview program.

2.2. Proposed method

The LVDT sensor measures the spring elongation, $u(x)$, defined as $u(x) = x_a - x - l_0$, where x_a, x, and l_0 represent the position of the arm, the position of the sled, and the spring elongation at rest, respectively. The fundamental principle of dynamics applied to the sled can be included in the expression of the coefficient of friction:

$$\mu = \frac{F_f}{F_N} = \frac{1}{m \cdot g}\left(F_p - m \cdot \frac{d^2x}{dt^2}\right) = \frac{k \cdot u}{m \cdot g} - \frac{1}{g}\left(\frac{d^2x_a}{dt^2} - \frac{d^2u}{dt^2}\right) \tag{1}$$

where F_f, F_p, m, g, and k represent the friction force, the pulling force applied by the arm on the sled, the mass of the sample–sled system, the standard gravity (g=9.81 m s^{-2}), and the spring stiffness of the spring, respectively. The second derivative of the elongation is noisy. Therefore, we identify a sixth order polynomial interpolation of the elongation for every measurement, as detailed in supplementary materials. The result of the interpolation is noted as $w(x)$. The difference between $w(x)$ and $u(x)$ is found to be lower than 0.1% which allows us replacing u by its estimation w. The arm is moving at a constant speed, its acceleration is therefore zero. We thus obtain from Eq. (1):

$$\mu = \frac{k \cdot w}{m \cdot g} + \frac{1}{g} \cdot \frac{d^2w}{dt^2} \tag{2}$$

In conclusion, the proposed method permits (i) the measurement of the pulling force applied by the arm on the sled, (ii) the calculation of the velocity of the sled, (iii) the calculation of the coefficients of friction, and (iv) the measurement of the acceleration of the sled.