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Friction-induced vibration and dynamic friction laws: Instability at positive friction–velocity-characteristic

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ABSTRACT

Sliding friction can lead to unstable vibration. This vibration can be unwanted, e.g. brake noise, or wanted, e.g. in violin strings. Several excitation mechanisms of friction-induced vibration are known, but still not all excitations observed in practice can be explained. Moreover, many experiments show that friction itself is highly dynamic and is by far more complex than a friction law with velocity-dependency implies.

We show how the stability of an oscillator sliding on a belt will change, if a dynamic friction law with inner variable is considered instead of a velocity-dependent coefficient of friction. Unstable vibration can even be found in the case of a positive velocity-dependency of the coefficient of friction.

1. Introduction

In tribological systems many mechanisms are known that lead to friction-induced vibration. It is widely accepted that a one-dimensional oscillator sliding on a belt can perform friction-induced vibration if the coefficient of friction decreases with increasing relative velocity between oscillator and belt. However, some cases are known in which a constant or even positive friction characteristic can excite vibration. These mechanisms are moving mass excitations [1], parameter excitations [2,3], mode-coupling [4,5], sprag-slip [6] or nonlinearities [7]. All these mechanisms require the consideration of additional mechanical degrees of freedom and cannot be modelled by only one mechanical degree of freedom. Moreover many approaches ignore that friction itself is highly dynamic and depends on sliding velocity, pressure, temperature and friction history itself. Some friction laws consider this by introducing inner degrees of freedom to the friction laws [8–11]. This dynamic friction will interact with a vibrating structure. For this interaction a simple velocity-dependent friction characteristic can fail in predicting the stability of the system vibration. In this work we investigate the impact of a friction law on an oscillator, where friction considers history, temperature and sliding velocity.

2. Model with a dynamic friction law

To investigate the influence of friction dynamics on a dynamic system, we apply a one-dimensional oscillator moving parallel to the sliding surface, as it has been studied multiple times. A dynamic friction law is considered in the contact between oscillator and belt, Fig. 1.

The oscillator position x is described with respect to an arbitrarily chosen reference position. To ensure continuous sliding and to avoid the stiction mode, the belt velocity $v>0$ is chosen sufficiently high $v>-\dot{x}$. So the equation of motion yields

$$\ddot{x}+\omega^2\cdot x=-n\mu. \quad (1)$$

Here $\omega>0$ is the angular eigenfrequency of the oscillator and $\mu>0$ is the coefficient of friction. The variable $n>0$ is a measure for the normal force between the oscillator and the belt, which can be obtained by the normal force divided by the mass of the oscillator. In the following derivation, damping is not introduced. Additional damping will have a simple stabilizing effect in the context of this investigation.

To model the dynamic friction behaviour, a friction law introduced by Ostermeyer [11] is applied. This friction law was derived from investigations of dynamic processes on the surfaces of sliding brake pad and brake disc. Here on at least one surface of the two sliding bodies a process of continuous growth and destruction of mesoscopic thin hard structures was observed. These structures consist of wear debris, which is compacted at positions of local maxima of temperature and pressure. The quantity and surface

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covering ratio of these thin hard structures have a direct impact on the coefficient of friction. This impact was cast into a macroscopic dynamic friction law [11], which is applied here in a linearized form:

$$\dot{\mu} = -a \cdot (\mu - \mu_0 - cT) \tag{2}$$

$$\dot{T} = -b \cdot (T - T_0 - nd(v + \dot{x})). \tag{3}$$

In this friction law, the coefficient of friction and the interface temperature T are described by coupled ordinary differential equations. The friction equation (2) allows a first-order lag behaviour of the coefficient, as it has been observed in experiments for example in [8,10,12]. The saturation time is determined by coefficient $a > 0$. This coefficient requires a positive value to avoid instability in the friction law. Here μ_0 is the reference coefficient of friction at $T=0$. Parameter c scales the influence of the interface temperature T on the coefficient of friction. The temperature behaviour, Eq. (3), can be approximated by a differential equation with time constant $b > 0$. T_0 is the ambient temperature and $d > 0$ scales the energy which is produced by friction with relative sliding velocity $v_{rel} = v + \dot{x}$. The friction law can be applied in sliding systems with mesoscopic thin hard structures, e.g. in brake, clutch or grinding systems, and beyond in more general frictional applications. Studies of the dynamics in this friction law and comparisons to measurements are published e.g. in [12,13].

The entire dynamic system is described by Eqs. (1)–(3) with different parameters. In this system of equations, the following parameters are positive:

$$a > 0$$
$$b > 0$$
$$d > 0$$
$$n > 0$$
$$\omega > 0. \tag{4}$$

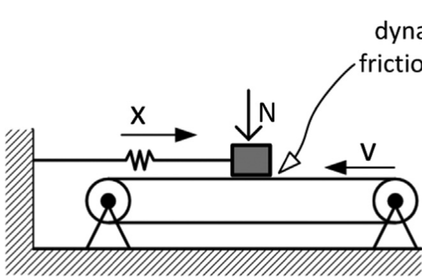

Fig. 1. Oscillator with dynamic friction law.

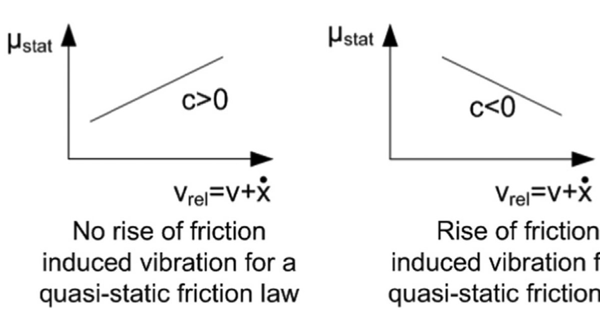

Fig. 2. Velocity-dependency of a quasi-static friction law and expected stability or instability of the system.

3. Results with a quasi-static friction law

In many tribological tests, the coefficient of friction is determined at different sliding velocities. In this kind of experiment, a constant sliding velocity v is chosen. After the measured coefficient of friction converged against a static value, this value is assumed as the coefficient of friction at a specific sliding velocity. This static coefficient of friction can be obtained from the dynamic friction law in Eqs. (2) and (3) for $\dot{\mu}_{stat} = \dot{T}_{stat} = 0$ and provides a static coefficient of friction of

$$\mu_{stat} = \mu_0 + cT_0 + cnd(v + \dot{x}). \tag{5}$$

By the choice of c and cnd every desired linearized dependency on temperature and sliding velocity can be described. This formulation is called "quasi-static" friction law, as it still contains a velocity-dependency, but only for the stationary solution of Eqs. (2) and (3). Inserting this equation into the oscillator equation and rearranging terms lead to

$$\ddot{x} + n^2 cd \cdot \dot{x} + \omega^2 \cdot x = -n(\mu_0 + c(T_0 + ndv)). \tag{6}$$

In the following, theory of stability at a point of rest is applied to describe the appearance of friction-induced vibration: "unstable" refers on the rise of vibration while "stable" describes a solution decaying to zero amplitudes. Obviously, solution (6) is stable for $c > 0$ and unstable for $c < 0$, as Fig. 2 underlines.

4. Results with dynamic friction law

As the static measurement of the coefficient of friction at a specific sliding velocity v ignores the transient change of friction, it can "hide" the dynamics of the friction law, which is described in Eqs. (2) and (3). This effect becomes significant, if a system vibrates with its eigenfrequencies. The vibration causes changes in the relative velocity in the sliding interface, which will in turn trigger different dynamics of the friction law. To investigate how this dynamics can lead to increasing vibrations, a stability analysis is carried out.

For stability evaluation, the friction law and the dynamics of the oscillators can be written in one state space formulation for small disturbances at the point of rest

$$\frac{d}{dt}\begin{bmatrix} \mu \\ T \\ x \\ \dot{x} \end{bmatrix} = \begin{bmatrix} -a & ac & 0 & 0 \\ 0 & -b & 0 & ndb \\ 0 & 0 & 0 & 1 \\ -n & 0 & -\omega^2 & 0 \end{bmatrix} \begin{bmatrix} \mu \\ T \\ x \\ \dot{x} \end{bmatrix} \tag{7}$$

and stability can be discussed in terms of eigenvalues. The four complex eigenvalues λ_i of the system

$$\underline{\dot{u}} = \underline{\underline{M}}\,\underline{u} \tag{8}$$

are the roots of the corresponding characteristic polynomial:

$$det\left(\underline{\underline{M}} - \lambda_i \underline{\underline{I}}\right) = 0. \tag{9}$$

To reduce the number of parameters to be evaluated, the system is transferred into a dimensionless form by introducing

$$\Lambda_i = \frac{\lambda_i}{\omega}$$
$$A = \frac{a}{\omega} > 0$$
$$B = \frac{b}{\omega} > 0$$
$$C = \frac{n^2 cd}{\omega} \quad \text{with } \mathrm{sign}(C) = \mathrm{sign}\,(c). \tag{10}$$

This is especially valuable, as in practical applications it is expensive to experimentally determine the different parameters in