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Determination of the limits of quasi-static/rigid and dynamic solutions for problems with frictional interfaces☆

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ABSTRACT

Frictional interfaces exhibit complex, nonlinear behaviour, and are often sources of energy dissipation, wear, and failure mechanisms. High fidelity models of a system with frictional interfaces, however, can be computationally intensive due to the nonlinearity. Thus, numerous techniques exist that each requires different assumptions for an analysis. One categorical divide in techniques is between quasi-static and dynamic analyses. These two phenomenologically different methods are compared in order to ascertain the regimes over which each of these methods is valid. Understanding of the extent of the inertial dominated and stiffness dominated regimes offers insight into the contribution of wave propagation effects to the system's response at the frictional interface, and determines the limits of applicability of each type of analysis.

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1. Introduction

One of the greatest remaining problems in structural mechanics is the modeling of frictional interfaces and their dissipative properties. Energy dissipation and frictional contact, though, is a topic germane to many branches of engineering. In particular, energy dissipation is central to the mechanics of jointed interfaces. These commonly are found as two surfaces in frictional contact as part of a built-up structure, such as in engines [1,2], automotive brakes [3], aerospace structures [4,5] and blade-disk assemblies [6,7], or bridges [8] amongst many other applications. These interfaces exhibit complex, nonlinear behaviour, and are often sources of not only energy dissipation, but also wear and potential failure mechanisms. Consequently, it is important to develop accurate models of frictional interfaces in order to understand the stresses, dynamics, and responses of a system under various loadings.

Due to the nonlinearity introduced by the frictional interface, numerical computation of the response of a system containing interfaces can be prohibitively expensive. As a result, numerous techniques have been developed in order to create pragmatic models of built-up structures. These techniques, though, each require different sets of assumptions for an analysis. One categorical divide in techniques is between quasi-static analyses and dynamic analyses. Quasi-static analyses such as [5,9–12] are restricted to frequency ranges in which wave propagation effects are negligible, but allows for a high fidelity treatment of the localized stress and displacement fields. That is, it is assumed that in most practical applications, the accelerations (and consequently the inertial terms) are small enough to be neglected, which leads to the quasi-static formulation [13]. Conversely, dynamic analyses such as [14–17,7] necessitate either large, impractical simulations that model the frictional interface in high fidelity, or, in most cases, simplified simulations that regularize the application and effects of friction as occurring in a discrete manner in exchange for computational efficiency. A natural consequence of this categorical divide in modeling approaches is the set of questions:

- What effect does regularization of the contact interface have on the predicted energy dissipation rates?
- At what frequency is the quasi-static assumption of negligible acceleration terms no longer valid?

1.1. The effect of regularization on energy dissipation

From the study of interfacial mechanics, the actual mechanics of stick-slip behaviour over an interface have significant consequences for predicting dissipative properties such as the coefficient of friction [6,18]. Regularization of the contact interface, i.e. the simplification of the kinematics at a contact interface to a single degree of freedom, can lose the effects of micro-slip behaviour, which generally leads to severe under-predictions of energy

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Nomenclature

A_j	cross-sectional area of rod j
c_j	normalization constant for the nth axial mode shape of rod j
E_j	Young's modulus of rod j
F_j	compressive axial force applied to the boundary of rod j
F_{yn}	effective axial force due to the superposition function
$F_{\Theta n}$	axial force due to the coupling with rotational displacement
G_j	shear modulus of rod j
J_j	second polar moment of area of rod j
L_j	length of rod j
N	number of modes used in the dynamic solution
p_0	applied axial pressure
r	radius of both rods in the quasi-static model
r_j	radius of rod j
T_j	internal torque in rod j
T_{Dmax}	the maximum interfacial torque for the dynamic model
T_{QSmax}	the maximum interfacial torque for the quasi-static model
T_{Un}	torque due to the coupling with axial displacement
$T_{\varphi n}$, $T_{\varphi jn}$	modal torque for the rotational displacement
t	time
U_j	total axial displacement of bar j
u_j	elastic displacement of bar j
x_j	axial position of bar j
y_j	superposition function for the axial displacement of rod j
α_{jn}, β_{jn}	coefficients for the torsional mode shapes
γ_{jn}	normalization constant for the nth torsional mode shape of rod j
η_n	temporal coefficient for the nth torsional mode
κ_{Aj}, κ_{Tj}	coupling coefficients in the axial and torsional equations of motion for rod j
κ_A, κ_T	non-dimensional coupling coefficients
λ_n	wave speed of the nth torsional mode
μ	coefficient of friction
ν_j	Poisson's ratio of rod j
ρ_j	density of rod j
Θ_j	total rotational displacement of rod j
θ_j	elastic rotational displacement of rod j
ϕ_{jn}	the nth mode shape of rod j
φ_j	superposition function for the radial displacement of rod j
σ_n	wave speed of the nth axial mode
τ_{max}	maximum shear stress at the interface
Υ_n	normalization constant for the nth torsional mode shape of rod j
Ω	boundary excitation
Ω_S	amplitude of boundary excitations with $S=E, A, B$, and C
ω_E	excitation frequency
ω_n	natural frequency of the nth torsional mode
ϖ_n	natural frequency of the nth axial mode
ξ_n	temporal coefficient for the nth axial mode
$(\cdot)^*$	denotes non-dimensional value of preceding variable

dissipation rates. Further, the distribution of pressure across the contact area is non-uniform as the real contact surfaces are rough and tend to develop clusters of concentrated contact patches [13] that are the primary source of interfacial friction [19]. For instance, even when the shear force is insufficient to cause sliding, the presence of partial slip regions can lead to the bulk motion of a body [20]. A conclusion, then, of quasi-static analyses is that modeling the contact interface in a discrete manner with a single Coulomb friction model is inadequate to model accurately all of the frictional effects in a real joint [19] (even though it may well describe the global behaviour [4]) since the partial slip behaviour that is not captured accurately is fundamental to precise predictions of energy dissipation and localized behaviour.

Quasi-static simulations, though, have several limitations. The principle limiting factor associated with modeling joints is the high computation cost required for simulations [17], which leads to the conclusion that if the full description of the joint interface is included in a dynamic simulation, then the computational cost would be prohibitively expensive. Additionally, dynamic effects due to wave propagation and resonances are observed to influence significantly the energy dissipation characteristics of a joint at high frequencies [8]. Thus, a key question in the dynamic analysis of jointed structures is: how can joints be modeled accurately and efficiently without resorting to a full representation of the interfacial mechanics? There have been a number of different methods proposed over the past century to address this question [21]. Historically, these methods have been somewhat ad hoc, and generally employ tuned linear models that are valid only for a specific range of excitation frequencies. In order to develop a higher fidelity approach to modeling joints, recent research focuses on developing reduced order constitutive models to model accurately the hypothesized behaviour of a jointed interface; these models, however, still regularize the interface to be a single discrete point. One model in particular, the Iwan model [22], shows promise as a more general representation of joint friction [19]. Furthermore, the Iwan model is able to represent accurately the energy dissipation characteristics calculated from a quasi-static model of an elastic rod on a frictional foundation [1], which implies that one potential avenue for research is the further development of these models to allow for high accuracy constitutive modeling in dynamic simulations without the high computational cost of faithfully modeling the entire interface. One major constraint of this approach, though, is that the model parameters still require tuning with experimental data in order to be predictive [23], which can be a challenging task in built-up structures [2]. Thus, with regard to the question of what is the effect of regularization of the contact interface on the predictions of the system's dynamics, energy dissipation, wear, and potential failure mechanisms, the answer is dependent on the validity of the constitutive model used to represent the contact interface [24].

1.2. The frequency range of validity for quasi-static models

In the absence of a predictive, reduced order, high fidelity constitutive model of joint interfaces, analysts are left to decide between using high fidelity quasi-static models or regularized dynamic models to study the dynamics, energy dissipation, wear, and potential failure mechanisms in a built-up structure. Central to this decision is the question: at what frequency is a quasi-static assumption no longer valid? This question is addressed by studying the system of Fig. 1, in which two cylinders are arranged end-to-end, in Section 2. The quasi-static solution for the inception of slip is developed in Section 2.1. From the dynamic perspective, this system is first considered for purely torsional motion in