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An approximate solution of oil film forces of turbulent finite length journal bearing

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ABSTRACT

An approximate analytical method is proposed for calculating oil film forces of turbulent finite length journal bearings. The dynamic 'π' oil film assumption is usually taken to determine oil film forces in the dynamic analysis of hydrodynamic journal bearing-rotor systems. However oil film field is not in the 'π' zone, i.e. the start position of the oil film is not 0, and the termination position of the oil film is not 'π'. Based on the variational principle, separation of variables is employed to obtain the pressure distribution in this paper. The pressure distribution of the infinitely long journal bearing model is taken as a circumferential separable function of the pressure distribution. The start and termination positions of oil film in the circumferential direction are determined by using the continuity condition. The axial separable function of the pressure distribution is obtained by the variational principle and the circumferential separable function. The results calculated by the proposed method are in good agreement with the oil film forces by the finite element method, and the computing cost is reduced greatly. Meanwhile, the influence of the parameters on the oil film forces is analyzed.

1. Introduction

The bearing-rotor system has been widely applied to rotary machinery. Safe running of the rotors supported by journal bearings is directly related to the security of the rotary machines. The instability of the bearing-rotor system has become an important problem in the application and development of the rotary machines undergoing higher speed and power, such as half speed whirl of the rotor induced by the oil film. The bearing-rotor system exhibits typical nonlinear system behaviors, and such nonlinearity arises generally from the oil film forces of the bearing. Therefore, the study on oil film forces of the bearing-rotor system has become important in nonlinear analysis of bearing-rotor system.

In many studies, nonlinear dynamic behaviors and stability of the bearing-rotor system are usually investigated based on the infinitely short bearing [1–6] or the infinitely long bearing model [7–10]. When the ratio of bearing length-to-diameter is very small, the oil film pressure distribution along the circumferential direction can be neglected based on the assumption of the infinitely short bearing model. On the other hand, the ratio of bearing length-to-diameter is very large in the infinitely long journal bearing model, and thus the pressure flow in axial direction can be omitted.

The models of the infinitely short bearing and the infinitely long bearing are very simple, and these bearing models cannot describe the actual bearing. In order to calculate the oil film forces accurately, several types of numerical methods for the hydrodynamic lubrication have been studied [11–14]. Zheng et al. [11] presented a Ritz model to calculate the oil film forces, which led to save computing time greatly. The presented model matched the free boundary by simply introducing a parameter. Xiao et al. [12] proposed a rapid and efficient algorithm for fluid forces based on the theory of variational inequality. The algorithm for solving the fluid forces and their Jacobis was transformed to solve a set of linear algebraic equations with tridiagonal coefficient matrices. Lu et al. [13] determined the oil film forces and their Jacobis simultaneously with compatible accuracy and without an increase in computing costs by the variational constraint approach. In order to save computing costs further, Lu et al. [14] calculated the oil film forces based on the database theory.

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Nomenclature

O_b	the center of the bearing
O_j	the center of the shaft journal
Θ	the deviation angle
Φ	the absolute circumferential coordinate of the bearing
φ	the circumferential coordinate of the bearing
f_r	oil film force in negative radial direction
F_r	dimensionless oil film force in negative radial direction
f_t	oil film force in negative tangential direction
F_t	dimensionless oil film force in negative tangential direction
f_x	oil film force in negative x direction
F_x	dimensionless oil film force in negative x direction
f_y	oil film force in negative y direction
F_y	dimensionless oil film force in negative y direction
h	the thickness of the oil film
H	the dimensionless thickness of the oil film
ω	the rotating speed of the rotor
R	the radius of the bearing
r	the radius of the shaft journal
ψ	the clearance ratio
d	the diameter of the bearing
mg	the load of the bearing
B	the width of the bearing
G_φ	turbulence factors
G_z	turbulence factors
ρ	the density of the lubricant
c	radial clearance
U	the circumferential velocity of the shaft journal
μ	the dynamic viscosity of the lubricant
Re	Reynolds number
e	eccentricity of the shaft journal
p	the pressure distribution of the oil film
P	the dimensionless oil film pressure
λ	the dimensionless axial coordinate
ε	the dimensionless eccentricity
τ	the dimensionless time
ε'	the dimensionless radial velocity
$\varepsilon\theta'$	the dimensionless tangential velocity
φ_s	the oil film start position in the circumferential direction
φ_c	the oil film termination position in the circumferential direction
S	the Sommerfeld number ($S=\mu\omega Br^3/(wc^2)$)
S'	the defined Sommerfeld number ($S'=1/w$, S' is used instead of S in this paper)

The numerical methods are accurate, but they have large computational expenses. In order to reduce computing time, many researchers determined the oil film forces by the approximate analytical methods [15–19]. Vignolo et al. [15] proposed an approximate analytical solution of the Reynolds equation for finite length journal bearings by means of the multi-parameters method, and the static oil film forces were obtained. Hirani et al. [16], and Bastani and Queiroz [17] applied different combinations of the results of short bearing and long bearing models to approximate the pressure distribution of finite length journal bearing under different conditions. Wang et al. [18] obtained the analytical solution of the oil film pressure distribution of finite length plain journal bearings based on dynamic Gümbel boundary conditions, i.e. dynamic 'π' oil film boundary conditions. Sfyris and Chasalevris [19] presented the expression of oil film pressure distribution using the method of separation of variables in an additive and a multiplicative form. A set of particular solutions of the Reynolds equation was added in the general solution of the homogeneous Reynolds equation, and then a closed form expression of oil film pressure distribution was formed. These researchers provided the approximate/exact analytical solutions of the oil film pressure distribution of the bearing-rotor system under laminar conditions.

In practice, the oil film of the bearings is under turbulent condition as the rotors run with high speed. Chang et al. [20], studied nonlinear dynamic behaviors of a rotor system supported by turbulent journal bearings. Based on the infinitely long bearing model, an analytical expression of oil film forces of the turbulent infinitely long bearing was obtained. Lo et al. [21] analyzed nonlinear dynamic behaviors of a flexible rotor system supported by turbulent journal bearings which considered the influence of coupling stress, and the infinitely short bearing model was employed. Based on the infinitely short bearing model, Wang and Khonsari [22] investigated the steady state and the dynamic characteristics of a rotor-bearing system with turbulent effects using the Hopf bifurcation theory. Wang et al. [23–25] have studied the model of complex turbulent lubrication. The lubrication performance of journal bearings with strong axial flow can be described by the proposed model efficiently under the working condition of high speed and heavy load. In addition to the presented model and its theory being advanced, the solving procedure is complicated and time consuming. Although the computational cost can be reduced further by the database method, the approximate analytical solution still cannot be obtained.

In this paper, the pressure distribution of the infinitely long journal bearing is taken as a circumferential separable function of pressure distribution. The start and termination positions of oil film in circumferential direction are determined by using the continuity and mass conservation conditions. The axial separable function of pressure distribution is obtained by the variational principle and the circumferential separable function. The approximate analytical expressions of oil film forces of turbulent finite length journal bearing are derived based on the variational principle and the method of separation of variables.

2. Oil film forces of turbulent finite length journal bearings

The turbulent finite length journal bearing model and its coordinates are shown in Fig. 1. Fig. 2 shows the cross section of the finite length journal bearing. In Figs. 1 and 2, O_b is the center of the bearing, O_j is the center of the shaft journal, θ is the deviation angle, Φ is the angle from negative y-axis direction to the oil film position along the clockwise direction as shown in Fig. 1, φ is the angle which goes from extension of O_jO_b to the oil film position as shown in Fig. 1, f_r and f_t are oil film forces in negative radial and tangential directions respectively, f_x and f_y are oil film forces in negative x and y directions respectively, h is the thickness of oil film, ω is the rotating speed of the rotor, R is the radius of the bearing, r is the radius of the shaft journal, w is the load of the bearing, B is the width of the bearing, and d is the diameter of the bearing.

If the lubricant is considered as an incompressible fluid, the Reynolds equation can be written as follows:

$$\frac{\partial}{\partial x}\left(\frac{G_\varphi h^3}{\mu}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(\frac{G_z h^3}{\mu}\frac{\partial p}{\partial z}\right)=\frac{U}{2}\frac{\partial h}{\partial x}+\frac{\partial h}{\partial t} \tag{1}$$