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From a compressible fluid model to new mass conserving cavitation algorithms

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ABSTRACT

This study investigates two algorithms proposed to solve a new cavitation model. This new cavitation model is based on a compressible Reynolds equation in which the density–pressure relation is obtained from a barotropic–isentropic assumption. It can be viewed as an approximation of the Jakobson–Floberg–Olsson/Elrod Adams cavitation model. Two algorithms are proposed to solve it. The first one is explicit and needs an important number of nodes. The second is implicit and can be used for steady-state and unsteady problems. Its implementation is easy and needs only minor modifications for a computer code in which cavitation is ignored. It can also be used to compute the solution of the usual J.F.O./E.A. model. Faster convergence is obtained using a relaxation parameter.

1. Introduction

Cavitation in lubrication is a complex process which has been the subject of numerous studies, both from the physical aspect and the numerical one [1]. The boundary conditions used to describe the cavitation region are usually a zero pressure gradient and a constant given value p_{cav} (the so called cavitation pressure). This constant value is often assumed on the whole cavitation area while the sign restriction $p > p_{cav}$ is applied on the non cavitated area. This model has often been called the Reynolds cavitation model. For a long time, Christopherson algorithm [2] has been widely used to describe the cavitation. One of its primary attributes is that it is easy to perform: It is a slight modification of the well-known Jacobi or Gauss Seidel algorithms used to solve the system of equations obtained by discretizing Reynolds equation using finite elements or finite difference methods: The only modification being the introduction of an additional line which modifies each computed term which is less than p_{cav} and puts it to this value during the iterative procedure. The main disadvantage of this model is that it is not a mass-conserving one. This feature can often be neglected in a lot of situations (plain journal bearing or slider in fully flooded situation) if the load or the attitude angle are the only operational parameters of interest. However if input mass flow values have to be considered, if starvation occurs or if roughness cannot be neglected, the Reynolds cavitation model must not be used [18]. Most of the works about mass conserving cavitation models are based on the Jakobsson–Floberg–Ollson (J.F.O.) [3] cavitation model and the Elrod Adams (E.A.)algorithm [4]. The basic idea is to describe the mass flow not only as a function of the pressure p but also of an another unknown θ which is the relative saturation (or fluid saturation) with the constraints

$$p \geq p_{cav}, \quad 0 \leq \theta \leq 1, \quad (p - p_{cav})(\theta - 1) = 0 \tag{1}$$

The model is a conservative one by construction. It is however mathematically much more complicated: hyperbolic in the cavitation area and elliptic in the non cavitated area.

The present proposed paper will address three aspects of the previously mentioned cavitation J.F.O/E.A. models:

- It is difficult to obtain them in a rigorous way using a thin film procedure, starting from a full 3 dimensional description. For example the E.A. model is obtained by a modification of a mass flow description which is only valid assuming a homogeneous 3-dimensional flow [4].
- All these models assume the fact that the pressure never falls below the cavitation pressure p_{cav}. However sub-ambient pressure loops have been observed as early as 1982 by Etsion and Ludwig [5] and Braun and Hendricks [6] one year later. Values of pressure as small as 0.07 MPA have been observed. The existence of such under-pressure can be neglected for heavily loaded devices. For light loaded devices however, the constraint $p > p_{cav}$ cannot be retained and previous models are not suitable. Moreover, inside the cavitation area, the variation of the density with respect to the pressure and the variation of the viscosity cannot be taken into account.
- Although it is possible to prove that the J.F.O./E.A model is a well posed problem [7], the computation of the solution is not easy. Elrod and Adams identify as a difficulty the fact that the

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Nomenclature

c_l (c_v) m/s sound velocity in the pure liquid region (pure vapor region)
h gap (m) (h_{max}, h_{min} maximal and minimal value of the gap)
p pressure (Pascal),
p_{vm},(p_{sat}) transition pressures between pure vapor-mixture regime (mixture-pure liquid regime)
p_{cav} cavitation region in JFO/EA model (Pa)
p_{inf} numerical parameter in the approximation of JFO/EA model (Pa)
p_{ext} boundary condition for the pressure (Pa)
t time (s)
u relative velocity of the upper surface of the device (m/s) in direction x_1
x_1,x_2 coordinates ($x=x_1$ for one-dimensional Reynolds equation)
L length of the device (m)
L_{cav} length of the cavitation region (test 4)
M number of nodes
W load
X normalized first coordinate: $X=x/L$
β Bulk parameter
μ_l (μ_v) pascal s-viscosity in the pure liquid region (pure vapor region)
$\rho_l(\rho_v)$ (kg/m^3) density of liquid at pressure p_{sat} (of vapor at pressure p_{vm})

relation θ–p in Eq. (1) is not one to one (for example the value $\theta=1$ is not associated to a unique value of the pressure). They modify this relation in the non cavitated area by introducing a small compressibility parameter. Numerous methods have been proposed to deal with both E.A and J.FO. models. Most of them are based on the E.A algorithm and its Vijayaraghvan–Keith improvement [8] by introducing various iterative process coupling pressure and saturation [9,10]. Some others used the characteristic methods to deal with the hyperbolic feature of the equation [11]. In some particular cases it is possible to consider a system of coupled ordinary differential equations instead of the partial differential equation [12]. Recently the Linear Complementarity numerical method has been used to solve the discretized problem [13]. These methods are not easy to implement, so that, despite their physical disturbing feature, Christopherson method is often used even today.

Recently, a thin film procedure starting from the compressible 3-dimensional Navier–Stokes equation with variable density and (dynamic) viscosity has been rigorously performed [14]. Assuming simplified properties (barotropic and isentropic assumptions)of the fluid, a Reynolds compressible equation is obtained which is very close to the E.A. model and improved it in some aspects:

- Variation of both density and viscosity in cavitated region and in "full film" region can be considered
- No constraint like $p > p_{cav}$ is introduced so that under-pressure can be obtained.

Detailed properties of this new model which will be called "fully compressible model" (FC) have been already given [15]. We discuss in this paper numerical algorithms to solve this model. We will show how a simplified numerical mass conserving algorithm can be deduced for the JFO–EA model. This algorithm is very close to the well-known Christopherson algorithm and so is very easy to implement.

2. New "fully compressible" fluid model

The Reynolds equation is written for a model device with a small gap $h(x_1,x_2)$ in which the upper part is fixed and the flat lower part has a constant velocity u along the x_1-main direction. Neglecting variation of the temperature, viscosity μ and density ρ are assumed to be only function of the pressure.

$$\frac{\partial}{\partial x_1}\left(\frac{h^3}{12\mu(\rho)}\rho(p)\ \frac{\partial p}{\partial x_1}\right)+\frac{\partial}{\partial x_2}\left(\frac{h^3}{12\mu(\rho)}\rho(p)\ \frac{\partial p}{\partial x_2}\right)=u/2\frac{\partial(\rho(p)h)}{\partial x_1}+\frac{\partial(\rho(p)h)}{\partial t} \tag{2}$$

Although this equation is written for transient situations, it will firstly be considered for a steady state situation, so neglecting the time derivative term. Dynamic aspects will be treated in Section 5.

The cavitation phenomenon is implicitly contained in the density–pressure relation in the context of the vaporous cavitation. Three distinct regimes are considered: one of pure vapor, one for pure liquid and one of mixture. The input data are: the velocities of the sound, c_v and c_l, the density ρ_v and ρ_l and the viscosities μ_v and μ_l in each of the pure regimes. From these data, it is possible to compute the transition pressure between these 3 regimes: p_{vm} between vapor and mixture and p_{sat} between mixture and liquid [15]

$$p_{vm}=\rho_v c_v^2 \tag{3a}$$

$$p_{sat}=\rho_v c_v^2-N\ \log\ \left(\frac{\rho_v^2 c_v^2}{\rho_l^2 c_l^2}\right) \tag{3b}$$

$$with\ \ N=\frac{\rho_v c_v^2 \rho_l c_l^2(\rho_v-\rho_l)}{\rho_v^2 c_v^2-\rho_l^2 c_l^2} \tag{3c}$$

It is convenient to introduce the void fraction α defined by:

$$\alpha=(\rho-\rho_l)/(\rho_v-\rho_l)$$

Pressure–density relations are:

$$p(\alpha)=c_v^2\rho \qquad if\ \ \rho<\rho_v \tag{4a}$$

$$p(\alpha)=P_{sat}+c_l^2\ (\rho-\rho_l) \qquad if\ \ \rho>\rho_l \tag{4b}$$

$$p(\alpha)=P_{sat}\ \ +N\ \log\ \left(\frac{\rho_v c_v^2\rho}{\rho_l(\rho_v c_v^2(1-\alpha)+\rho_l c_l^2\alpha)}\right) \qquad if\ \ \rho_v\le\rho\le\rho_l \tag{4c}$$

In the mixture region, viscosity–density relation can be chosen as ([16,17]):

$$\mu(\alpha)=\alpha\mu_v+(1-\alpha)\mu_l \quad \text{Dukler assumption} \tag{5}$$

or

$$1/\mu(\alpha)=M(\alpha)/\mu_v+(1-M(\alpha))/\mu_l \quad \text{McAdams assumption} \tag{6}$$