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Reduced order finite element model for elastohydrodynamic lubrication: Circular contacts

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ABSTRACT

This paper presents an extension of the reduced order finite element model to the case of circular elastohydrodynamic lubricated (EHL) contacts under isothermal Newtonian considerations. The line contact model was developed and validated in a previous work (Advances in Engineering Software, 2013; 56:51 – 62). The model is based on a finite element discretization of the EHL equations: Reynolds, linear elasticity and load balance with a reduced order model for the linear elasticity part. All equations are solved simultaneously in a fully-coupled framework using a damped-Newton procedure allowing fast convergence rates for the global solution. This model combines fast convergence rates, reduced memory requirements and negligible model reduction errors compared to the full model which makes it an attractive tool for EHL contact performance prediction.

1. Introduction

The development of a fast, accurate and memory efficient elastohydrodynamic lubrication (EHL) solver has been an ongoing process in the Tribology community over the last five decades. The numerical modeling of an EHL contact is a challenging problem with many difficulties. These are mainly attributed to two features: first, a strong coupling between several physical problems: hydrodynamics, linear elasticity, heat dissipation and second, the extremely nonlinear nature of the problem related to the strong pressure–temperature dependence of the transport properties of common lubricants. This has led to the development of several models over the years; each attempting to overcome these difficulties without compromising accuracy, robustness and performance.

One of the first attempts to a comprehensive modelling of the isothermal EHL problem including the elastic deformation of the solid components along with the pressure dependence of the transport properties of the lubricant was presented in the pioneering work of Dowson and Higginson [1] later followed by a more comprehensive study by Hamrock and Dowson [2]. These works were based on a finite difference discretization of the EHL equations that were solved using a semi-system approach. That is, equations were solved separately and an iterative procedure was established between their respective solutions, leading to a slow convergence rate of the overall procedure due to the loss of information occurring in the weak coupling process. In addition, the use of finite differences restricted the approach to regular structured meshing, which led to unnecessarily large matrix systems. The elastic deformation of the solid components was computed using an integral approach assuming a half-space configuration. The latter was associated to a very large computational overhead as the calculation of the elastic deformation at every discretization point involved a numerical integration over the entire computational domain. A major improvement came with the incorporation of multigrid techniques to these models by Lubrecht et al. [3]. This allowed a significant improvement in the convergence rates as well as the large computational times associated to regular structured grids. Another milestone in the improvement of these models came later with the application of multigrid techniques to the integral calculation of the elastic deformation of the solid components by Venner [4] who also introduced the line relaxation scheme which allowed an extension of the range of application of these models to include high loads with Hertzian pressures reaching up to several Gigapascals.

Another approach that, by virtue of its nature, involves very fast convergence rates is the full-system approach in which all EHL equations are solved simultaneously preventing any loss of information in the coupling process. One of the first works to use such an approach is that of Rhode and Oh [5,6] who solved the EHL problem as one integro-differential equation using a finite element (FEM) discretization and a nonlinear Newton-like resolution. However, the integral part of the problem which connects every point of the discretization domain to all other points leads to a full Jacobian matrix which inversion requires a large computational overhead. A similar approach was later provided by Houpert and Hamrock [7] for the line contact case and extended to the case of

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Nomenclature

A_1, A_2	Modified WLF model constant parameters
B_1, B_2	Modified WLF model constant parameters
C_1, C_2	Modified WLF model constant parameters
E_i	Young's modulus of solid body i
E_{eq}	Equivalent Young's modulus
F	External load
H	Dimensionless film thickness
H_0	Dimensionless film thickness constant parameter
L	Dimensionless Moes material properties parameter
M	Dimensionless Moes load parameter
N_{2D}	Number of dof in the 2D hydrodynamic problem
N_{3D}	Number of dof in the 3D linear elasticity problem
N_{dof}	Total number of dof of the full model
$\tilde{N}_{dof}$	Total number of dof of the reduced model
N_m	Number of basis functions employed in the reduced model
P	Dimensionless pressure
Pe	Peclet number
R	Cylindrical roller radius
S_P	Pressure solution space
S_U	Elastic deflection solution space
T_0	Ambient temperature
$T_g(0)$	Lubricant's ambient pressure glass transition temperature
U	Elastic displacement vector
X,Y,Z	Dimensionless space coordinates
a	Hertzian contact radius
p	Pressure
p_h	Hertzian pressure
u,v,w	x, y and z components of the elastic displacement vector
u_i	Surface velocity of solid body i
u_m	Mean entrainment speed
α^*	Equivalent pressure-viscosity coefficient
μ_g	Lubricant's viscosity at glass transition temperature
μ_R	Lubricant's reference viscosity
$\overline{\mu}$	Lubricant's dimensionless viscosity
ν_i	Poisson's coefficient of solid body i
ν_{eq}	Equivalent Poisson's coefficient
φ^i	Basis function i
$\overline{\rho}$	Lubricant's dimensionless density
ρ_R	Lubricant's reference density

Subscripts

e	Elastic
h	Hydrodynamic
l	Load balance

Dimensionless Parameters

$X=\frac{x}{a}$ $Y=\frac{y}{a}$ $Z=\frac{z}{a}$ $P=\frac{p}{p_h}$ $\overline{\rho}=\frac{\rho}{\rho_R}$ $\overline{\mu}=\frac{\mu}{\mu_R}$ $H=\frac{hR}{a^2}$

elliptical contacts by Hsiao et al. [8]. Besides the difficulties associated to the inversion of a dense Jacobian matrix, all these models also involved difficulties in the implementation of the cavitation boundary condition at the exit of the contact because of the simultaneous solution of all pressure updates. In addition, the range of application was limited to light and moderately loaded contacts. More recently, Holmes et al. [9] introduced a new model using a full-system approach where the elastic part is based on the differential deflection method introduced earlier by Evans and Hughes [10]. This method consists in deriving a differential equation based on the half-space approximation in which the differential operator has a more localized nature. That is, every discretization point is only affected by its neighbouring points leading to a sparse Jacobian matrix. However, for the point contact case, the system matrix still had a large bandwidth, requiring a special iterative technique for an efficient resolution of the coupled equations.

More recently, Habchi et al. [11,12] introduced a finite element full-system approach for the solution of the EHL problem in which the elastic part of the problem is based on a classical linear elasticity approach. This model provided a remedy to the difficulties related to the full-system approach. In fact, the free boundary arising at the exit of the contact is treated in a straight forward manner by the use of the penalty method introduced by Wu [13]. The use of the finite element method in which every discretization point is only connected to neighbouring points belonging to the same element(s) led to a sparse Jacobian matrix. In addition, the meshing process was no longer restricted to regular and structured grids, which led to considerable reduction in the total number of unknowns and thus the overall size of the global matrix system. As for highly loaded contacts, the authors introduced special stabilized finite element formulations allowing an extension of the range of application of the method to include high loads with Hertzian pressures up to several Gigapascals. With all these difficulties being overcome, this model profited from the fast convergence properties of a full-system approach combined with a Newton-like resolution. The model was validated against existing ones and its performance was shown to be at least similar to state-of-the-art models. Nevertheless, a major improvement was still possible. In fact, the linear elasticity equations employed in computing the elastic deformation of the solid components are not restricted to the surface of these solids but rather extend to the subsurface domains. However, for the solution of the EHL problem, only the surface deformations of the solids are required. Therefore, a large number of degrees of freedom (dofs) are computed in vain. In order to improve this part of the model, Habchi and Issa [14] introduced a novel EHL-oriented model order reduction technique for the computation of EHD elastic deformations and applied it to the solution of isothermal Newtonian line contacts. It consists in defining the elastic deformation of the solid components as a linear combination of carefully selected and pre-computed EHL deformations called "basis functions". With this new technique, the elastic deformation of the solid components is obtained using less than 30 degrees of freedom (dofs). Therefore, not only memory requirements of this newly developed "reduced model" were lighter but also an order of magnitude reduction in cpu times was obtained with respect to the "full model". In addition, model reduction errors on central and minimum film thicknesses were shown to be of the order of only 1‰.

The current paper offers an extension of the "reduced model" to the case of isothermal Newtonian circular contacts for which both cpu times and memory requirements are significantly larger than for line contacts. First, the different EHL equations are reminded in Section 2. In Section 3, the numerical model developed in this work is described in details. Section 4 provides an investigation of the numerical performance of the "reduced model" for the case of circular contacts. Finally, Section 5 offers a conclusion to this work.