Short Communication

Nonlinear stability boundary of journal bearing systems operating with non-Newtonian couple stress fluids

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A B S T R A C T

The non-Newtonian effects on the nonlinear stability boundary of short journal bearings are investigated through the transient nonlinear analysis. Two coupled nonlinear equations are solved by using the fourth-order Runge-Kutta method. According to the results, there exists a nonlinear stability boundary within the clearance circle. Any initial positions of the shaft center outside of this boundary would yield an unstable trajectory, even though the bearing should be stable in accordance with the linear stability theory. The non-Newtonian effects provide a larger stability boundary within the clearance circle as compared to the bearing lubricated with a Newtonian fluid.

1. Introduction

Understanding of bearing characteristics is necessary for the design of machine elements in engineering applications. Fundamental analyses of journal bearings can be observed on the steady state problems, for example, by Pinkus and Sternlicht [1], Williams [2], and Capone et al. [3]. The load capacity, friction parameter and attitude angle are presented through the variation of the eccentricity ratio. However, fluid film lubricated journal bearings are apt to undergo a sort of self-excited oscillation or known as whirl instability. When the amplitude of whirl oscillation grows too large, it may endanger the safe manipulation of bearing systems. Knowing the nature of whirl instability depending upon the operating conditions is helpful for bearing selection and designing. By linearization of the nonlinear equations of motion, the linear stability analysis is made, for example, by Holmes [4] and Lund [5]. Below the stability threshold speed, the motion of the journal under a small disturbance about its equilibrium positions is found to be stable. In addition, the weakly nonlinear motion of journal bearings is analyzed using the method of multiple scales by Gardner et al. [6], and using the Hopf bifurcation theory by Lin [7]. It is found that the journal bearing may exhibit subcritical or supercritical bifurcation behaviors for running speeds near the neutral stability curve. In order to get further insights into the nonlinear behavior of journal bearings, studies of tracking the locus of the shaft center by using a transient method are presented by Crooijmans et al. [8]. It is found that the rotor suddenly jumps to a large periodic orbit as the rotor speed exceeds the linear stability threshold speed for high values and for low values of the modified Sommerfeld number. Further nonlinear analyses of the shaft path are contributed by Khonsari and Chang [9]. According to their results, the initial conditions of the shaft play an important role in picturing the transient characteristics of the shaft orbit. Although the running speed below the stability threshold speed yields linearly stable behavior according to the linearized stability theory, there exists a stable boundary within the clearance circle outside of which any initial positions will predict an unstable orbit.

In recent years, applications of non-Newtonian fluids as lubricants have received a great interest. Common lubricants displaying non-Newtonian behaviors are, for example, the lubricants blended with various types of additives, polymer-thickened oils, synthetic oils, and synovial fluids. From the experimental evidence of Oliver [10], the use of polymer-thickened oils can enhance the load capacity and reduce the friction coefficient of journal bearings. According to the experimental research by Scott and Suntiwattana [11], a Newtonian lubricant blended with suitable additives can reduce the wear and friction between rubbing surfaces. Since the classical Newtonian hypothesis is not able to simulate the flow behavior of these kinds of non-Newtonian fluids, a micro-continuum theory has been developed by Stokes [12]. The Stokes micro-continuum theory can describe the peculiar flow behavior of fluids which contain substructures and allow for the presence of...
body couples and couple stresses. The effects of couple stresses can be regarded as a result of the action of one part of a fluid element on its neighborhood. It is important for applications of pumping fluids, for example, the animal bloods, bio-fluids, liquid crystals and synthetic lubricants. Applying the Stokes micro-continuum theory of non-Newtonian couple stress fluid model, Shehawey and Mekheimer [13] and Pandey and Chaube [14] investigated the problems of peristaltic transport flow. Walicki and Walicka [15] and Bujurke and Kudenatti [16] studied the squeeze film mechanisms with reference to lubrication mechanism of synovial joints. Naduvinanamia et al. [17], Elsharkawy and AL-Fadhalah [18] and Lin et al. [19] analyzed the performance characteristics of two approaching surfaces. For the rotor bearing system, the non-Newtonian influences of couple stresses have also been investigated on the steady performances by Lin [20], the linear stability characteristics by Lin [21] and the weakly nonlinear bifurcation behavior by Lin [22]. The couple stress effects result in an increased load capacity and a reduced friction parameter [20]. For general applied load numbers, the journal bearings with couple stress fluids under small disturbances are shown to be more stable than the traditional Newtonian-lubricant case [21]. It is also found that for a fixed value of the bearing parameter, increasing the non-Newtonian couple stress parameter shifts the super-critical bifurcation to a lower steady eccentricity-ratio region [22]. Although a nonlinear stability boundary within the clearance circle for journal bearings with a Newtonian fluid has been obtained by Khonsari and Chang [9], the nonlinear analysis for the bearing with a non-Newtonian fluid is still absent. Therefore, a further study is motivated.

Based upon the micro-continuum of Stokes [12], the effects of non-Newtonian couple stress on the nonlinear stability boundary of short journal bearings are mainly concerned in the present study. Two coupled nonlinear second-order differential equations governing the motion of journal center are solved by using the fourth-order Runge-Kutta method. Although the running speed below the stability threshold speed yields linearly stable behavior according to the linearized stability theory, unstable orbits for the non-Newtonian fluid lubricated bearing depending upon the initial positions are illustrated through the transient nonlinear analysis. In addition, the influences of non-Newtonian fluids on the size of the nonlinear stability boundary within the clearance circle are investigated through the variation of the non-Newtonian couple stress parameter.

2. Analysis

Fig. 1 describes the physical configuration of a journal bearing system. The journal of R is rotating with an angular velocity \( \omega \) within the bearing housing. The film thickness is \( h^* = C + e \cos \theta \), where \( C \) is the maximum clearance, \( e \) is the eccentricity, \( \theta = x^*/R \) is the circumferential coordinate. The lubricant in the film region is taken to be a non-Newtonian incompressible couple stress fluid of Strokes [12]. Under the usual assumptions of thin-film lubrication theory, the non-Newtonian dynamic Reynolds equation governing the dynamic film pressure \( p^* \) can be expressed as Lin [21],

\[
\begin{align*}
\frac{\partial}{\partial x^*} \left[ h^* - 12 \rho h^* + 24 \Omega h^* \tanh \left( \frac{h^*}{2\rho} \right) \right] \frac{\partial p^*}{\partial x^*} &= 6 \rho \left( \omega^* - 2 \frac{d \omega^*}{d t^*} \right) + 2 \frac{d \omega^*}{d t^*} \\
I^* &= \left( \frac{\rho}{\mu} \right)^{1/2}
\end{align*}
\]

where \( \varphi \) is the attitude angle, \( t^* \) is the time, \( \mu \) is the lubricant viscosity, \( \eta \) is a new material constant with the dimension of momentum and is responsible for the couple stress property. As the value of \( I^* \) approaches zero, the non-Newtonian dynamic Reynolds equation reduces to the classical Reynolds equation for the case with Newtonian lubricant of Gardner et al. [6]. In addition, the equations of motions of the journal in the vertical and horizontal directions are described, respectively, by

\[
\sum F_x = m \frac{d^2 X^*}{d t^*} = F_x^* \sin \varphi + W^*
\]

\[
\sum F_y = m \frac{d^2 Y^*}{d t^*} = F_y^* \cos \varphi + W^* - F_y^* \sin \varphi
\]

In these equations, \( m \) is the rotor mass, \( W^* \) is the load of the bearing, and \( F_x^* \) and \( F_y^* \) denote the dynamic film forces along the \( \varepsilon \) and \( \varphi \) directions respectively, where \( \varepsilon = e/C \) defines the eccentricity ratio. The film pressure can be solved from the dynamic Reynolds equation (1). Integrating the dynamic film pressure yields the dynamic film forces. Introduce the non-dimensional variables and parameters as follows.

\[
X = \frac{X^*}{C}, \quad Y = \frac{Y^*}{C}, \quad t = \omega^* t^*
\]

\[
F_x = \frac{F_x^*}{W^*}, \quad F_y = \frac{F_y^*}{W^*}, \quad h = \frac{h^*}{C}
\]

\[
\omega = \sqrt{\frac{m C}{W^* \rho^*}}, \quad S_m = \frac{\mu \omega^* R^3}{4 W^* C^2}, \quad l = \frac{I^*}{C}
\]

In these definitions, \( \omega \) is the non-dimensional angular running speed, \( S_m \) is the modified Sommerfeld number, and \( l \) denotes the couple stress parameter characterizing the non-Newtonian influences on the bearing performances. Then the equations of motions of the journal can be expressed in a non-dimensional form.

\[
\frac{d^2 X}{d t^2} = \ddot{e} - \ddot{\varphi}^2 = \frac{1}{\omega^2} (F_x \cos \varphi - F_y \sin \varphi + 1)
\]

\[
\frac{d^2 Y}{d t^2} = \ddot{\varphi} + 2 \dot{\varphi} \dot{\varphi} = \frac{1}{\omega^2} (F_y \sin \varphi + F_x \cos \varphi)
\]

where \( \ddot{e} = d^2 e/d t^2, \ddot{\varphi} = d^2 \varphi/d t^2, \dot{e} = d e/d t, \dot{\varphi} = d \varphi/d t, \) and

\[
F_x = -2 S_m \int_0^{1/2} (1 - 2 \varphi \xi - e \sin \theta + 2 \xi \cos \theta) \cos \theta d \theta
\]

\[
F_y = -2 S_m \int_0^{1/2} (1 - 2 \varphi \xi - e \sin \theta + 2 \xi \cos \theta) \sin \theta d \theta
\]

For the bearing running under the steady state, the steady attitude angle \( \varphi_s \) corresponding to each value of the steady