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Fluid film lubrication in the presence of cavitation: a mass-conserving two-dimensional formulation for compressible, piezoviscous and non-Newtonian fluids

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ABSTRACT

A mass-conserving formulation of the Reynolds equation has been recently proposed by some of the authors to deal with cavitation in lubricated contacts [1]. This formulation, based on the mathematical derivation of a linear complementarity problem (LCP), overcomes the drawbacks previously associated with the use of such complementarity formulations for the solution of cavitation problems in which reformation of the liquid film occurs. In the present paper, the methodology favoured in [1], already successfully applied to solve textured bearing and squeeze problems in the presence of cavitation in a one dimensional domain for incompressible fluids, has been extended to include the effects of fluid compressibility, piezoviscosity and the non-Newtonian fluid behaviour and it has been also applied to the analysis of two dimensional problems. The evolution of the cavitated region and the contact pressure distribution are studied for a number of different configurations which can be considered as relevant benchmarks.

In particular, some of the results obtained with the proposed scheme are critically analysed and compared with the predictions obtained using alternative formulations, including full CFD calculations. The stability of the proposed algorithm and its flexibility in terms of implementation of different models for compressibility, piezoviscosity and non-Newtonian behaviour are highlighted.

1. Introduction

Reynolds equation is commonly used to describe lubrication problems as an alternative to the more complex full Navier–Stokes equations. The assumptions the Reynolds equation is based on are fully satisfied in the analysis of the majority of the lubricated contact problems commonly found in practical applications. These assumptions are, namely, that both the ratio of the film thickness to the contact length and the Reynolds number are small.

Cavitation may occur due to the development of low pressures within the fluid film. Various formulations have been proposed in order to correctly simulate this phenomenon. In the cavitated regions the mechanical properties of the fluid vary significantly. Jakobsson and Floberg [2] proposed a mass conserving algorithm capable of analysing lubricant films in the presence of cavitation. The algorithm described in [2] uses ad-hoc equations to locate the cavitation boundaries, setting a fixed pressure value in the non-active regions while solving the Reynolds equation within the active counterparts. Elrod and Adams [3] first developed a cavitation algorithm that uses a single equation within the whole domain and does not require to explicitly locate the cavitation boundaries. They introduced a switch function, $g(p)$, to suppress the Poiseuille flow in the non-active regions. In particular, $g(p)$ equals one where the pressure is greater than the cavitation pressure, whereas it is null ($g(p)=0$) otherwise. In this way the Poiseuille term of the Reynolds equation can be artificially suppressed in the cavitation region. This model has been extensively employed in the analysis of lubrication problems and it has been successfully validated by comparison with experimental evidences for both dynamic transient and EHL problems, e.g. [4,5]. Moreover, starting from the pioneering works of Elrod [3,6], further algorithms have been developed that take into account the compressibility of the lubricant, e.g. Vijayaraghavan and Keith [7], Venner and Bos [8] and Sahlin et al. [9].

While in the formulation proposed by Elrod a clear separation between the cavitated and active regions in considered, other models exist in which the cavitation phenomenon is deduced from the solution of the constitutive equations of the lubricant, so that a detailed description of the mixture characteristics in the cavitated region is required. In such cases, an equivalent homogeneous

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compressible fluid is assumed and a smooth pressure–density relation is used in the entire domain, e.g. [10,11].

Attempts have been made in the past to solve the problem of determining the active and cavitated film regions using the concept of complementarity. In particular, the possibility of studying the free boundary problems related to cavitation in terms of variational inequality as described by Lewy and Stampacchia [12] was first noticed by Laratta and Marzulli [13] and then subsequently investigated by Rohde and McAllister [14] and Cimatti [15]. An interesting physical interpretation of the assumptions related to cavitation treated as a complementarity problem is presented by Strozzi [16]. Furthermore, Kostreva [17] and Oh [18] and Oh and Goenka [19] extended the problem of cavitation from hydrodynamic lubrication to elastohydrodynamic lubrication. They solved the problem of determining the active and cavitated film regions using the concept of complementarity. However, these classical methods based on a complementarity formulation do not ensure the conservation of mass. This undesired characteristic is due to the fact that such algorithms solve the Reynolds equation within the whole domain assuming a constant lubricant density. While this assumption can be reasonably accepted within the active regions, in the non-active regions the density varies in both space and time. Giacopini et al. [1] explained how assuming constant fluid density within the whole domain leads to an incorrect detection of film reformation. In the literature, additional mass-conserving methods exist that merge variational inequalities and JFO theory, [20]. A commonly shared result is that ensuring the mass continuity is mandatory to correctly predict the film rupture and reformation, especially where cavitation and reformation occur several times (e.g. studies of rough contacts [21], textured surfaces [22–24] and dynamically loaded journal bearings [25]). The algorithm proposed in [1] ensures the conservation of mass within the whole domain, employing a complementarity formulation of the lubrication problem in the presence of cavitation based on a newly defined set of complementarity variables.

Nevertheless, the work presented in [1] considers incompressible and isoviscous fluids, thus providing good results only for low contact pressures, where the density and viscosity variations in the active regions are negligible. As a consequence, this model is not suitable, at least in its original formulation, to study lubrication problems in many modern practical applications. In fact, the increasing quest for enhanced performance, the severity of operating conditions to which modern lubricated contacts can be subjected, and a constant need for more accurate predictions, have been responsible for the development of more complex formulations that take into account several lubricant behaviours not compatible with the classical Reynolds equation alone. In particular, at high pressures and in the presence of high sliding speeds, the compressibility of the fluid, as well as the piezoviscosity and the non-Newtonian behaviour can no longer be neglected. In the past decades, different formulations have been proposed, which overcome the limitation introduced by the hypothesis of incompressibility and isoviscosity. Although various authors in recent years studied the lubricant film behaviour employing the full Navier–Stokes equations in CFD solvers [26,27], Reynolds-based approaches maintain great importance and practical utility due to their simpler formulations, that lead to usually faster and less cpu-time consuming implementations and provide equal accuracy for most of the scenarios in which the change in fluid properties through film-thickness are negligible.

In the present paper, a novel formulation of the Reynolds equation for compressible, piezoviscous and shear-thinning fluids in the presence of cavitation is presented in terms of complementarity. A brief explanation of the complementarity formulation adopted is first reported together with a description of the different models employed to introduce compressibility, piezoviscosity and shear-thinning in the formulation. Then, a number of numerical examples are provided that focus on the comparison of the results obtained using the solver developed by the authors with (i) newly derived analytical solutions, (ii) alternative methods available in literature and (iii) CFD simulations. A detailed description of the implementation of the developed methodology in the finite element (FE) framework is reported in Appendix A.

2. Formulation

2.1. Complementarity formulation

In this section, a complementarity formulation of a compressible, piezoviscous and shear-thinning fluid is presented. In the definition of a complementarity algorithm, two aspects must be considered: (i) the identification of the complementary variables and (ii) the definition of the functional connection that relates them. The complementarity variables adopted here are the same as those proposed in [1], namely the pressure, p, and the void fraction, r, which is defined as

$$r=1-\frac{\rho}{\rho_p}, \tag{1}$$

where ρ is the density of the mixture of oil and gases and ρ_p the density of the lubricant at the given pressure p.

The Reynolds equation in one dimension for unsteady and compressible flows is

$$\frac{\partial}{\partial x}\left[\frac{\rho h^3}{6\mu}\frac{\partial p}{\partial x}\right]-2\frac{\partial}{\partial t}[\rho h]-U\frac{\partial}{\partial x}[\rho h]=0, \tag{2}$$

where h is the film thickness, U the entrainment speed (i.e ($U1+U2$)), and μ the fluid viscosity.

Recasting Eq. (2) in terms of p and r one obtains (see [1])

$$\frac{\partial}{\partial x}\left[\frac{h^3}{6\mu}\frac{\partial p}{\partial x}\right]-\frac{\partial}{\partial x}\left[r\frac{h^3}{6\mu}\frac{\partial p}{\partial x}\right]-2\frac{\partial h}{\partial t}+2\frac{\partial}{\partial t}[rh]-U\frac{\partial h}{\partial x}+U\frac{\partial}{\partial x}[rh]=0. \tag{3}$$

This equation is valid both in the full film region (active region) and in the cavitated (non-active) region.

The problem described by Eq. (3) can thus be formulated in terms of a linear complementarity problem (LCP), the complementarity variables being p and r

$$p\geq 0$$

$$r\geq 0$$

$$p\cdot r=0. \tag{4}$$

For a compressible fluid, density is a function of pressure and it can be explicitly expressed by the general formula

$$\rho_p=\rho_c f(p), \tag{5}$$

where ρ_c is the density at the cavitation pressure and $f(p)$ is the functional connection between ρ_p and p.

The complementarity variable r can then be rewritten as

$$r=1-\frac{\rho}{\rho_c f(p)}. \tag{6}$$

By substitution of Eq. (6) into Eq. (2) and normalising over ρ_c, one obtains

$$\frac{\partial}{\partial x}\left[f(p)(1-r)\frac{h^3}{6\mu}\frac{\partial p}{\partial x}\right]-2\frac{\partial}{\partial t}[f(p)(1-r)h]-U\frac{\partial}{\partial x}[f(p)(1-r)h]=0. \tag{7}$$